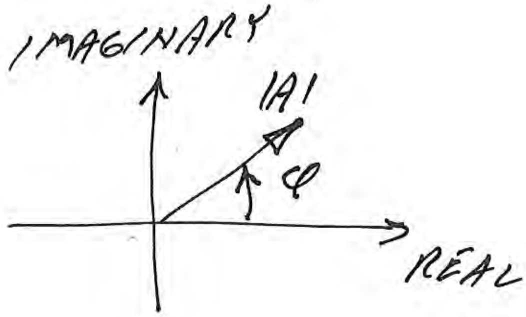
PHASORS

WE REPRESENT A POINT IN THE COMPLEX PLANE AS A VECTOR



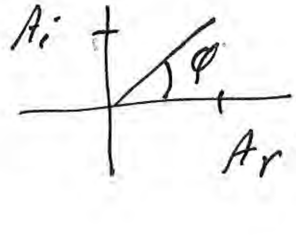
THIS VECTOR HAS REAL & IMAGINARY COMPONENTS

$$A = A_r + jA_i$$

$$A^* = A_r - jA_i$$

EQUIVALENTLY WE CAN WRITE

$$A = |A| e^{j\varphi} \quad ; \quad \varphi = \tan^{-1}\left[\frac{\Im m(A)}{\Re e(A)}\right]$$



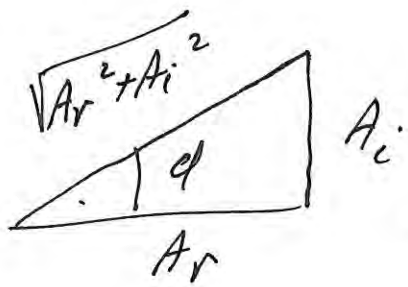
$$|A| = \sqrt{A \cdot A^*}$$

$$= \sqrt{A_r^2 + A_i^2}$$

THIS EQUIVALENCE CAN BE SEEN USING EULER'S IDENTITY

$$|A| e^{j\varphi} = \sqrt{A_r^2 + A_i^2}\,(\cos\varphi + j\sin\varphi)$$

BUT WHAT IS $\cos\varphi = \cos(\tan^{-1} A_i/A_r)$?



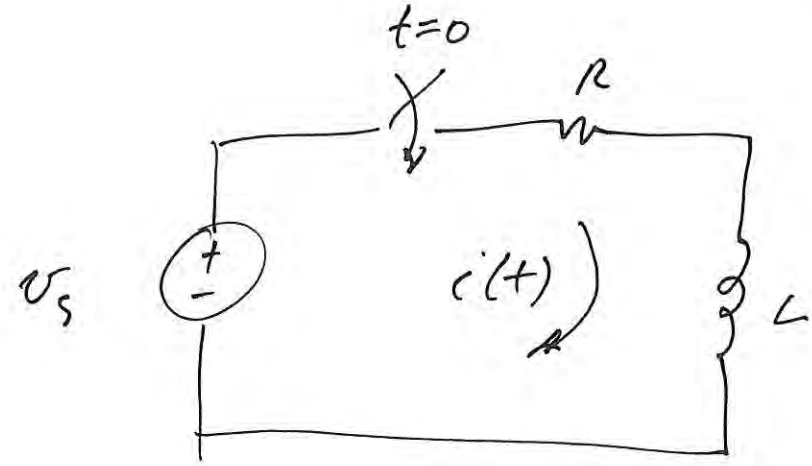
$$\cos\varphi = \frac{A_r}{\sqrt{A_r^2 + A_i^2}}$$

$$\sin\varphi = \frac{A_i}{\sqrt{A_r^2 + A_i^2}}$$

$$\therefore |A| e^{j\varphi} = \sqrt{A_r^2 + A_i^2}\left(\frac{A_r}{\sqrt{\quad}} + j\frac{A_i}{\sqrt{\quad}}\right)$$

$$= A_r + jA_i$$

Q.E.D.



SECTION 9.2
PG 311

FIG 9.5

KVL AFTER SWITCH IS THROWN

$$v_s - R\,i(t) - L\frac{di(t)}{dt} = 0$$

HERE $v_s = V_m \cos(\omega t + \varphi)$

D.E. IS $$R\,i(t) + L\frac{di(t)}{dt} = V_m \cos(\omega t + \varphi)$$

SINCE SWITCHED, KNOW SOL'N OF FORM

$$i(t) = i_h(t) + i_p(t)$$

$i_h(t)$ ← HOMOGENEOUS SOL'N.

$i_p(t)$ ← PARTICULAR SOL'N.

KNOW: $i_h(t) = Ae^{-t/\tau}$, $\tau = L/R$

IN PAST PROBLEMS i_p WAS CONST. WHY?

HERE THE PARTICULAR SOL'N IS A FUNCTION OF TIME. WHY?

WE KNOW PARTICULAR SOL'N HAS GENERAL FORM OF FORCING FUNCTION.

DOES THIS SUGGEST $i_p(t) = K\cos(\omega t + \varphi)$?

EASIER WAY

$$i_p(t) = Re\{B e^{j(\omega t + \varphi)}\}$$

↑ (B) CAN BE COMPLEX

APPROACH: IGNORE $Re\{\ldots\}$ BUT TAKE REAL PART OF FINAL ANSWER.

LET $i_p(t) = B e^{j(\omega t + \varphi)}$

WHEN ALL DONE WITH ALGEBRAIC MANIPULATIONS, KEEP ONLY REAL PART

USE $i_p(t) = Be^{j(\omega t+\varphi)}$

IN D.E.:

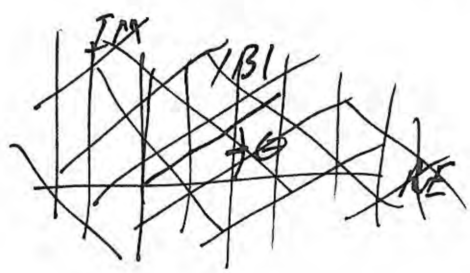
$$RBe^{j(\omega t+\varphi)} + Lj\omega Be^{j(\omega t+\varphi)} = V_m e^{j(\omega t+\varphi)}$$

$$B(R+j\omega L)e^{j(\omega t+\varphi)} = V_m e^{j(\omega t+\varphi)}$$

$$[B(R+j\omega L)-V_m]e^{j(\omega t+\varphi)} = 0$$

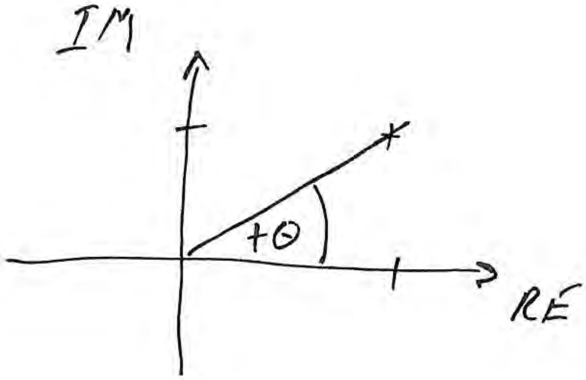
$$\therefore B = \frac{V_m}{R+j\omega L} \quad \text{(NOTE COMPLEX)}$$

WRITE $B = |B|e^{-j\theta}$, FIND $|B|$, & θ



$$B = \frac{V_m}{R+j\omega L}\,\frac{R-j\omega L}{R-j\omega L} = V_m\frac{(R-j\omega L)}{R^2+(\omega L)^2}$$

$$= \frac{V_m\sqrt{R^2+(\omega L)^2}\,e^{-j\theta}}{R^2+(\omega L)^2} = \frac{V_m}{\sqrt{R^2+(\omega L)^2}}e^{-j\theta}$$



$$+\theta = \tan^{-1}\frac{\omega L}{R}$$

$$i_p(t) = \mathrm{Re}\left\{ B e^{j(\omega t + \varphi)} \right\}$$

$$= \mathrm{Re}\left\{ \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} e^{-j\theta} e^{j(\omega t + \varphi)} \right\}$$

$$i_p(t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t + \varphi - \theta)$$

$$i(t) = i_n(t) + i_p(t)$$

$$= A e^{-t/\tau} + \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t + \varphi - \theta)$$

$$i(0) = 0 = A + \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \cos(\varphi - \theta)$$

$$i(t) = \frac{-V_m}{\sqrt{R^2+(\omega L)^2}} \cos(\phi-\theta) e^{-t/\tau}$$

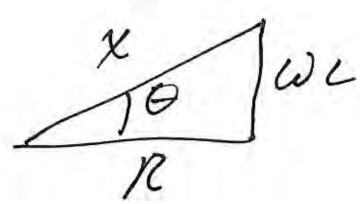
$$+ \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \cos(\omega t+\phi-\theta)$$

$$B = \frac{V_m}{\sqrt{(R)^2+(\omega L)^2}} e^{-j\theta}, \quad \theta = \tan^{-1}\frac{\omega L}{R}$$

CAN YOU SHOW THIS IS EQUAL TO

$$\frac{V_m}{R+j\omega L} \; ?$$

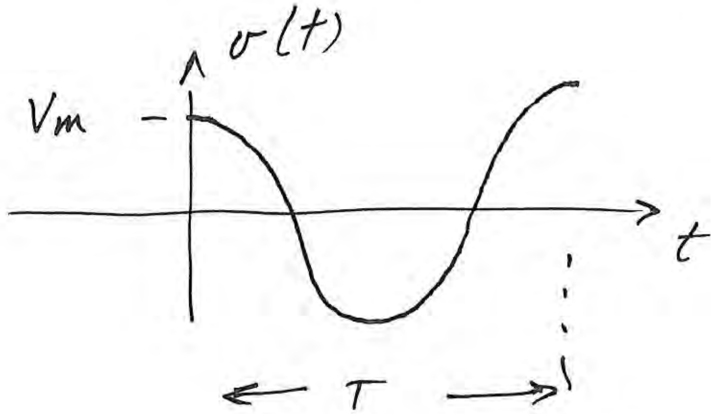
USE $e^{-j\theta} = \cos\theta - j\sin\theta$



FIND x

SINUSOIDAL SOURCES

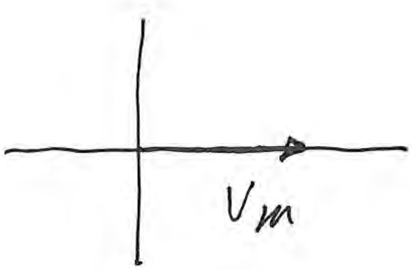
$$v(t) = V_m \cos(\omega t)$$



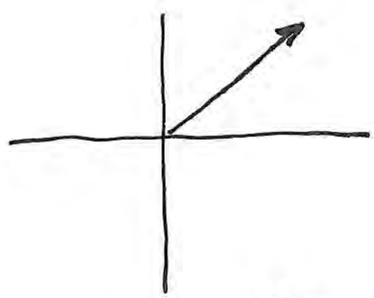
T = PERIOD : $v(t) = V_m \cos(2\pi f t)$

$T = 1/f$: $v(t) = V_m \cos\left(2\pi \frac{t}{T}\right)$

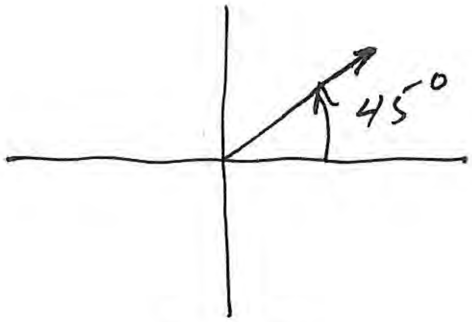
WHEN t GOES THROUGH A TIME EQUAL TO PERIOD, PHASE CHANGES BY 2π RADIANS OR $360°$



$t=0$

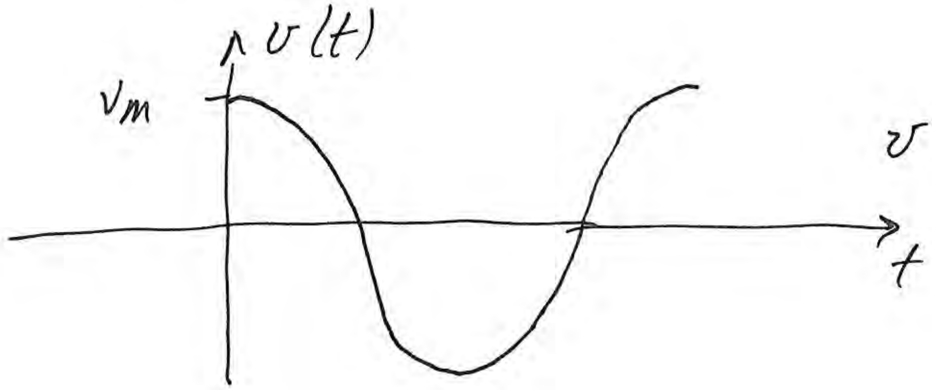


$t = T/8$



$\frac{T}{8} \times \frac{360^\circ}{T} = 45^\circ$

$t = T/8$



$v(t) = V_m \cos(\omega t)$

WHERE IS $v(t) = V_m$ (MAX VALUE)

ANS: AT $t = 0$

(ACTUALLY WHEN ARGUMENT

OF COS IS ZERO

COS(...))

WHAT IF WE ADD A PHASE?

$$v(t) = V_m \cos(\omega t + \varphi)$$

WHERE IS MAX?

ANS: WHERE $\omega t + \varphi = 0$

$$\text{OR } t = \frac{-\varphi}{\omega} = \frac{-\varphi}{2\pi f} = \frac{-\varphi}{2\pi} T$$

WITH $[\varphi]$ = RADIANS & $[T]$ = SEC,

$[t]$ = SEC

WHAT IF φ IS IN DEGREES?, φ°

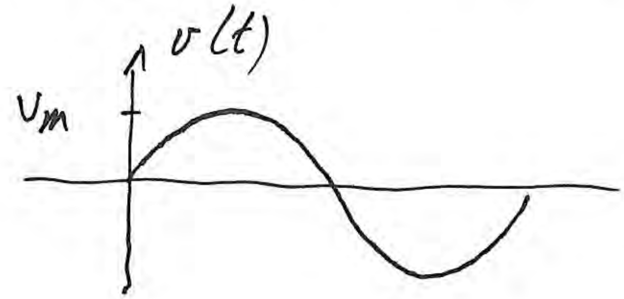
ANS: MAX IS AT $t = \frac{-\varphi^\circ}{360^\circ} T$

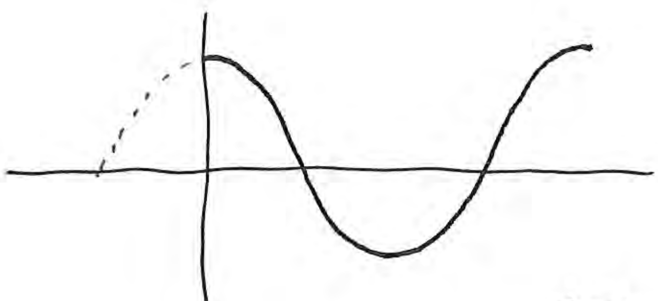
NOTE SIGN ON t

⇒ ADDING PHASE SHIFTS WAVEFORM TO THE LEFT

SAY $v(t) = V_m \sin(\omega t)$

HOW DO WE REPRESENT THIS AS A COS?





$V_m \cos(\omega t)$

REMEMBER: SUBTRACTING PHASE SHIFTS TO THE RIGHT

HOW FAR DO WE NEED TO SHIFT

$V_m \cos(\omega t)$

TO THE RIGHT TO MAKE IT LOOK LIKE $V_m \sin(\omega t)$?

ANS: <u>90°</u>

$\therefore \quad V_m \sin(\omega t) = V_m \cos(\omega t - 90°)$

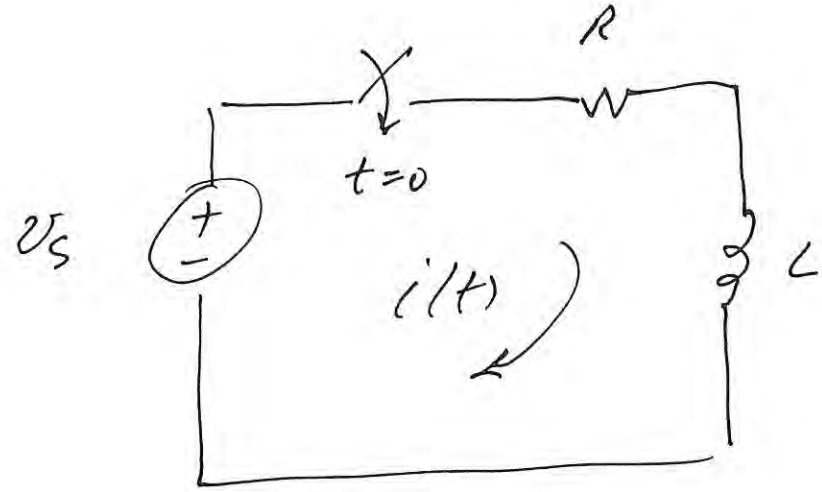
CONCLUSION: GIVEN A SIGNAL

$$V_m \sin(\omega t + \theta)$$

WE CAN WRITE THIS AS THE EQUIVALENT COSINE

$$V_m \cos(\omega t + \theta - 90^\circ)$$

WHERE MIGHT WE MAKE USE OF THIS TRANSLATION ?



WHAT IF $v_s = V_m \sin(\omega t + \varphi)$?

RMS VALUES

$$V_{RMS} = \left\{ \frac{1}{T} \int_{t_0}^{t_0+T} v^2(t)\,dt \right\}^{1/2}$$

$$= \left\{ \frac{1}{T} \int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t+\varphi)\,dt \right\}^{1/2}$$

NOTE INTEGRATION OVER PERIOD

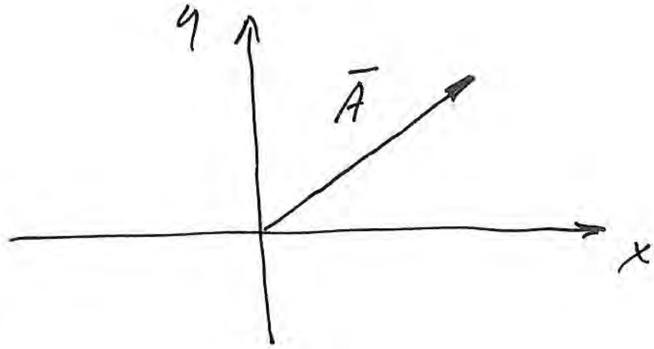
THIS DEF VALID FOR ANY t_0

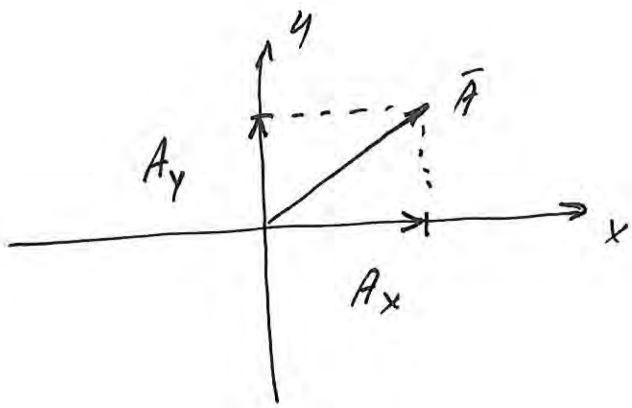
$$V_{RMS} = \left\{ \frac{1}{T} \; V_m^2 \frac{T}{2} \right\}^{1/2}$$

$$V_{RMS} = V_m/\sqrt{2}$$

ONLY FOR SINUSOIDES !

FOOTNOTE



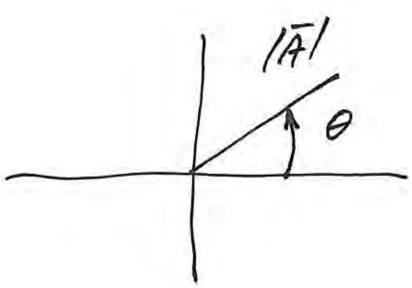


$\bar{A} = \hat{x} A_x + \hat{y} A_y$ $(\hat{x}, \hat{y})$ UNIT VECTORS

CARTESIAN

CAN ALSO CONSIDER POLAR FORM

$$\bar{A} = |\bar{A}| \angle \theta \quad ; \quad \theta = \tan^{-1}(A_y / A_x)$$

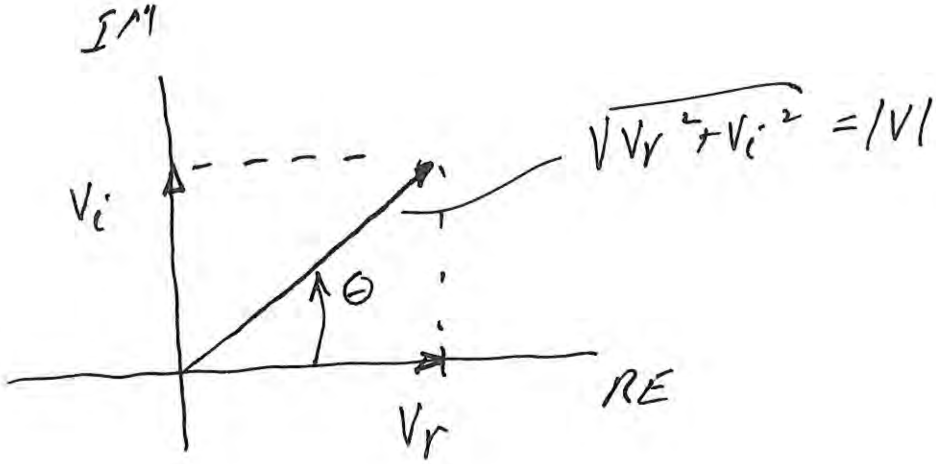


PHASORS... AGAIN

$$V_r + jV_i = \sqrt{V_r^2 + V_i^2}\, e^{j\theta}$$

$$\theta = TAN^{-1}(V_i/V_r)$$

CARTESIAN FORM   POLAR FORM



EULER'S IDENTITY $e^{j\theta} = \cos\theta + j\sin\theta$

$$|V|e^{j\theta} = |V|\cos\theta + j|V|\sin\theta$$

$$= |V|\frac{V_r}{|V|} + j|V|\frac{V_i}{|V|} = V_r + jV_i$$

SIGNAL REPRESENTATION

CONVENTION: $v(t) = V_m \cos(\omega t + \varphi)$

RESULT: $v(t) = V_m \, \mathrm{Re}\{ e^{j(\omega t + \varphi)} \}$

$$= \mathrm{Re}\{ V_m e^{j(\omega t + \varphi)} \}$$

$$= \mathrm{Re}\{ V_m e^{j\varphi} e^{j\omega t} \}$$

($V_m e^{j\varphi}$:) COMPLEX NUMBER REPRESENTING AMPLITUDE AND PHASE OF SIGNAL

WE CALL $V_m e^{j\varphi}$ THE PHASOR REPRESENTATION

OR "PHASOR TRANSFORM"

OF THE SIGNAL;

$$\bar{V} = V_m e^{j\varphi} \equiv \mathcal{P}\{ V_m \cos(\omega t + \varphi) \}$$

($\bar{V}$:) REPRESENTED IN TEXT IN BOLD NOTATION

$$\bar{V} = V_m e^{j\varphi} \equiv \underbrace{\mathcal{P}\{V_m \cos(\omega t + \varphi)\}}$$

"PHASOR TRANSFORM"
OF SIGNAL, $V_m \cos(\omega t + \varphi)$

NOTE $\bar{V} = V_m \cos\varphi + j V_m \sin\varphi$

WE OFTEN MAKE USE OF NOTATION

$$e^{j\varphi} = \angle\varphi^\circ \qquad \angle\varphi^\circ$$

$$\bar{V} = V_m \angle\varphi^\circ$$

WHAT ABOUT INVERSE TRANSFORM?

$$\mathcal{P}\{V_m \cos(\omega t+\varphi)\} = V_m e^{j\varphi}$$

$\mathcal{P}^{-1}$ TO BOTH SIDES

$$\mathcal{P}^{-1}\{\mathcal{P}\{V_m \cos(\omega t+\varphi)\}\} = \mathcal{P}^{-1}\{V_m e^{j\varphi}\}$$

$$V_m \cos(\omega t+\varphi) = \mathcal{P}^{-1}\{V_m e^{j\varphi}\}$$

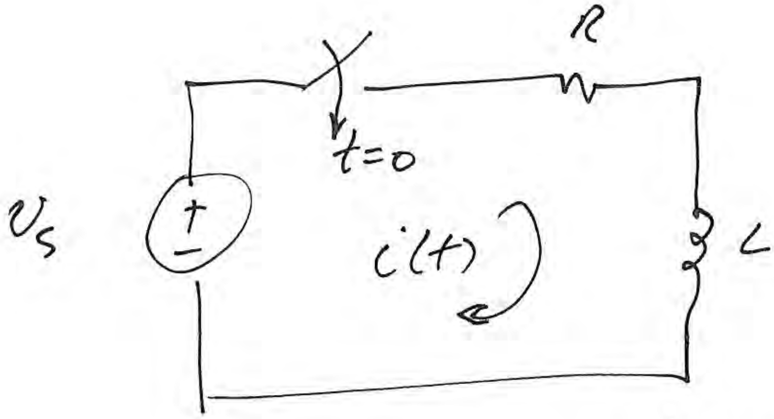
WHAT'S THIS? ↗

ANS: $\mathcal{P}^{-1}\{V_m e^{j\varphi}\} = Re\{\underbrace{V_m e^{j\varphi}}_{\bar{V}} e^{j\omega t}\}$

$$= Re\{V_m e^{j(\omega t+\varphi)}\}$$

$$V_m \cos(\omega t+\varphi) \overset{\mathcal{P}}{\longleftrightarrow} V_m e^{j\varphi}$$

TIMEDOMAIN — FREQUENCY DOMAIN



$$v_s - Ri(t) - L\frac{di(t)}{dt} = 0$$

$$v_s = V_m \cos(\omega t + \varphi)$$

RECALL S.S. SOLUTION IS OF FORM

$$i_{ss}(t) = \mathrm{Re}\{I_m e^{j\beta} e^{j\omega t}\}$$

$$\mathrm{Re}\{R I_m e^{j\beta} e^{j\omega t}\} + \mathrm{Re}\{j\omega L I_m e^{j\beta} e^{j\omega t}\}$$

$$= \mathrm{Re}\{V_m e^{j\varphi} e^{j\omega t}\}.$$

$$\mathrm{Re}\{(j\omega L + R) I_m e^{j\beta} e^{j\omega t}\} = \mathrm{Re}\{V_m e^{j\varphi} e^{j\omega t}\}$$

MORE SIMPLY

$$(j\omega L + R) I_m e^{j\beta} e^{j\omega t} = V_m e^{j\varphi} e^{j\omega t}$$

(TAKE REAL PART LATER)

KEEP IN MIND WHAT WE'RE SOLVING FOR...

$$(j\omega L + R) I_m e^{j\beta} = V_m e^{j\varphi}$$

$$I_m e^{j\beta} = \frac{V_m}{j\omega L + R} e^{j\varphi}$$

$$= \frac{V_m}{\sqrt{(\omega L)^2 + R^2}} e^{-j\theta} e^{j\varphi}$$

I_m

$$I_m e^{j\beta} = \frac{V_m}{\sqrt{(\omega L)^2 + R^2}} e^{j(\varphi - \theta)}$$

$$P^{-1}\{I_m e^{j\beta}\} = \frac{V_m}{\sqrt{(\omega L)^2 + R^2}} \cos(\omega t + \varphi - \theta)$$

$$\theta = \tan^{-1}(\omega L / R)$$

$$v(t) = V_m \cos(\omega t + \varphi)$$

PHASOR TRANSFORM

$$\bar{V} \equiv \mathcal{P}\{V_m \cos(\omega t + \varphi)\} = V_m e^{j\varphi}$$

NOTE: $$V_m \cos(\omega t + \varphi) = \mathrm{Re}\{V_m e^{j(\omega t + \varphi)}\}$$

$$= \mathrm{Re}\{V_m e^{j\varphi} e^{j\omega t}\}$$

$$= \mathrm{Re}\{\bar{V} e^{j\omega t}\}$$

$\Longrightarrow$ PHASOR TRANSFORM DESCRIBES THE SIGNAL COMPONENT AT THE ~~COMPLEX~~ FREQUENCY ~~$e^{j\omega t}$~~ ω

INVERSE PHASOR TRANSFORM

$$\mathcal{P}^{-1}\{\bar{V}\} = \mathcal{P}^{-1}\{V_m e^{j\varphi}\}$$

$$\mathcal{P}^{-1}\{V_m e^{j\varphi}\} = \mathrm{Re}\{V_m e^{j\varphi} e^{j\omega t}\}$$

$$= \mathrm{Re}\{V_m e^{j(\omega t+\varphi)}\}$$

$$= V_m \cos(\omega t + \varphi)$$

$$\mathcal{P}^{-1}\{\bar{V}\} = V_m \cos(\omega t + \varphi)$$

↑

$\bar{V} = V_m e^{j\varphi}$

$$V_m \cos(\omega t + \varphi) \overset{P}{\longleftrightarrow} V_m e^{j\varphi}$$

TIME DOMAIN FREQUENCY DOMAIN

✓ASSESSMENT PROBLEMS

Objective 1—Understand phasor concepts and be able to perform a phasor transform and an inverse phasor transform

9.1 Find the phasor transform of each trigonometric function:

a) $v = 170 \cos(377t - 40^\circ)$ V.

b) $i = 10 \sin(1000t + 20^\circ)$ A.

c) $i = [5 \cos(\omega t + 36.87^\circ) + 10 \cos(\omega t - 53.13^\circ)]$ A.

d) $v = [300 \cos(20{,}000\pi t + 45^\circ) - 100 \sin(20{,}000\pi t + 30^\circ)]$ mV.

Answer: (a) $170\underline{/-40^\circ}$ V;

(b) $10\underline{/-70^\circ}$ A;

(c) $11.18\underline{/-26.57^\circ}$ A;

(d) $339.90\underline{/61.51^\circ}$ mV.

9.2 Find the time-domain expression corresponding to each phasor:

a) $\mathbf{V} = 18.6\underline{/-54^\circ}$ V.

b) $\mathbf{I} = (20\underline{/45^\circ} - 50\underline{/-30^\circ})$ mA.

c) $\mathbf{V} = (20 + j80 - 30\underline{/15^\circ})$ V.

Answer: (a) $18.6 \cos(\omega t - 54^\circ)$ V;

(b) $48.81 \cos(\omega t + 126.68^\circ)$ mA;

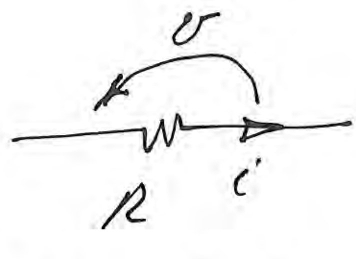
(c) $72.79 \cos(\omega t + 97.08^\circ)$ V.

NOTE: Also try Chapter Problem 9.11.

HOW DO WE USE THE CONCEPT OF A PHASOR TRANSFORM?

$\bar{V}$ - $\bar{I}$ RELATIONSHIPS

RESISTOR



(RECALL PASSIVE SIGN CONVENTION)

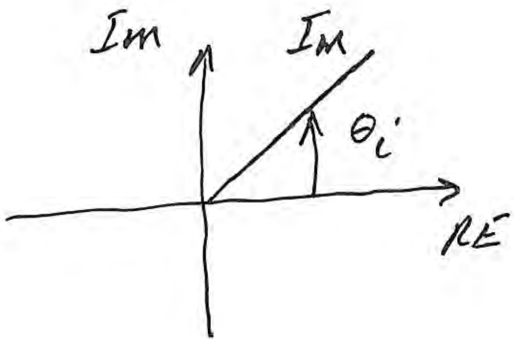
$$i = I_m \cos(\omega t + \theta_i)$$

$$v = R I_m \cos(\omega t + \theta_i)$$

↑ PHASE ANGLE OF CURRENT

$$\mathcal{P}\{v\} \equiv \bar{V} = R I_m e^{j\theta_i} = R\bar{I}$$

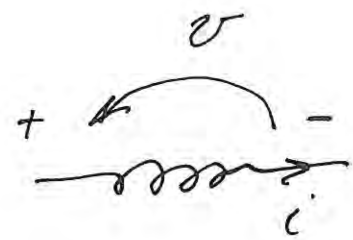
$$\boxed{\bar{V} = R\bar{I}}$$



$\bar{V}$ & $\bar{I}$ ARE IN PHASE

WHY?

INDUCTORS



$$v = L \frac{di}{dt}$$

WITH $i = I_m \cos(\omega t + \theta_i)$

$$v = L \frac{d}{dt} I_m \cos(\omega t + \theta_i)$$

$$= -\omega L I_m \sin(\omega t + \theta_i)$$

BY CONVENTION, WE NEED TO EXPRESS IN TERMS OF COSINE

WHY?

RECALL: SHIFT TO RIGHT BY SUBTRACTING PHASE

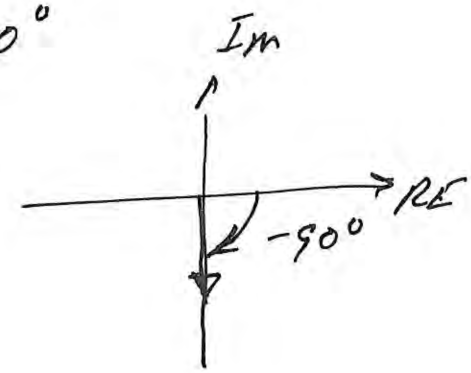
$$v = -\omega L I_m \cos(\omega t + \theta_i - 90°)$$

$$P\{v\} = -\omega L I_m e^{j(\theta_i - 90°)}$$

$$\mathcal{P}\{v\} = \bar{V} = -\omega L e^{-j90^\circ} \underbrace{I_m e^{j\theta_i}}_{\bar{I}}$$

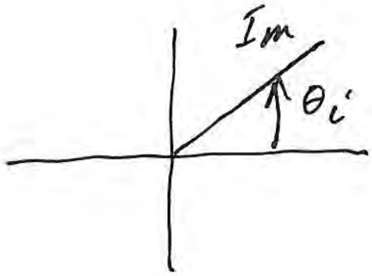
WHAT IS e^{-j90°

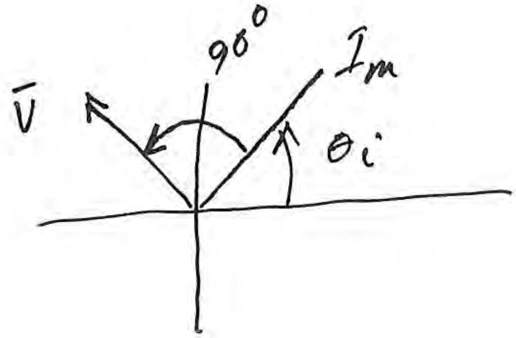
$$e^{-j90^\circ} = -j$$



$$\boxed{\bar{V} = j\omega L \bar{I}}$$

NOTE $j\omega L = \omega L e^{j90^\circ}$

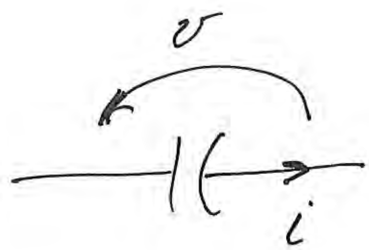




NOTE $\bar{V}$ & $\bar{I}$ OUT OF PHASE BY 90°

DOES VOLTAGE LEAD OR LAG?

CAPACITORS



$$i = C\frac{dv}{dt}$$

WITH $v = V_m \cos(\omega t + \theta_v)$

↑ PHASE ANGLE OF VOLTAGE

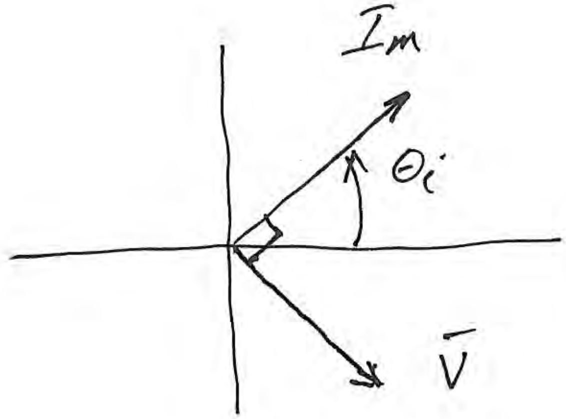
$$i = -\omega C V_m \sin(\omega t + \theta_v)$$

AS BEFORE,

$$\bar{I} = j\omega C \bar{V}$$

$$\boxed{\bar{V} = \frac{1}{j\omega C}\bar{I}}$$

NOTE $\frac{1}{j\omega C} = \frac{1}{\omega C} e^{-j90°}$



NOTE $\bar{V}$ & $\bar{I}$ OUT OF PHASE BY 90°

DOES VOLTAGE LEAD OR LAG ?

RESISTOR $\bar{V} = R\bar{I}$

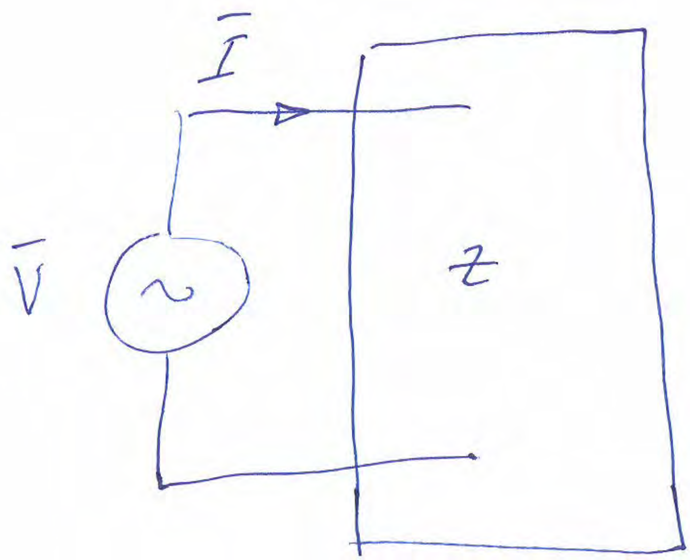
INDUCTOR $\bar{V} = j\omega L\bar{I}$

CAPACITOR $\bar{V} = \frac{1}{j\omega C}\bar{I}$

ALL OF FORM

$$\bar{V} = Z\bar{I}$$

"IMPEDANCE"



FOR IDEAL RESISTORS, INDUCTORS, CAPACITORS, Z IS EITHER COMPLETELY REAL OR PURELY IMAGINARY.

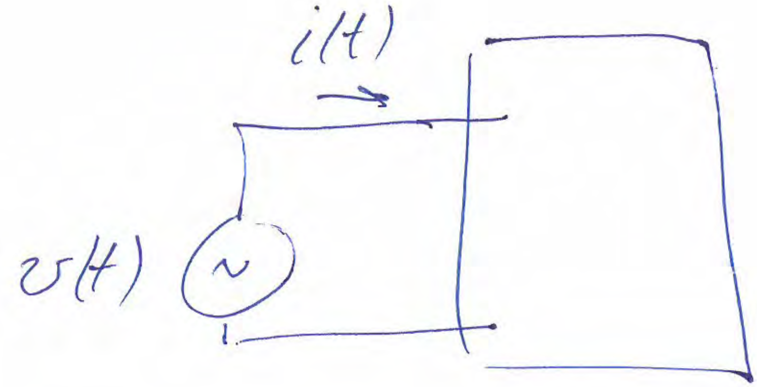
FOR A GENERAL CIRCUIT, Z IS COMPLEX.

$$\mathrm{Im}\{Z\} \equiv \text{"REACTANCE"} = X$$

FOR RESISTOR, REACTANCE $= 0$

FOR INDUCTOR, REACTANCE $= \omega L$

FOR CAPACITOR, REACTANCE $= -1/\omega C$



$$v(t) = V_m \cos(\omega t + \varphi)$$

$$i(t) = I_m \cos(\omega t + \varphi + \theta)$$

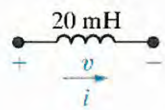
WHAT IS THE IMPEDANCE?

WHAT IS THE REACTANCE?

✓ASSESSMENT PROBLEMS

Objective 2—Be able to transform a circuit with a sinusoidal source into the frequency domain using phasor concepts

9.3 The current in the 20 mH inductor is $10 \cos(10{,}000t + 30^\circ)$ mA. Calculate (a) the inductive reactance; (b) the impedance of the inductor; (c) the phasor voltage **V**; and (d) the steady-state expression for $v(t)$.



Answer: (a) 200 Ω;

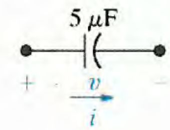
(b) $j200$ Ω;

(c) $2\angle 120^\circ$ V;

(d) $2 \cos(10{,}000t + 120^\circ)$ V.

NOTE: Also try Chapter Problems 9.12 and 9.13.

9.4 The voltage across the terminals of the 5 μF capacitor is $30 \cos(4000t + 25^\circ)$ V. Calculate (a) the capacitive reactance; (b) the impedance of the capacitor; (c) the phasor current **I**; and (d) the steady-state expression for $i(t)$.



Answer: (a) −50 Ω;

(b) $-j50$ Ω;

(c) $0.6\angle 115^\circ$ A;

(d) $0.6 \cos(4000t + 115^\circ)$ A.

SPECIFICATION OF SINUSOIDAL SOURCE

$SINE(V_{OFFSET}, V_m, f, t_D, \theta, \varphi)$

t_D ↑ DELAY θ ↑ DAMPING FACTOR ($1/s$)

EXAMPLE: WANT $20\cos(800t + 25°)$

USE $SINE(0, 20, 127.3, 0, 0, 115)$

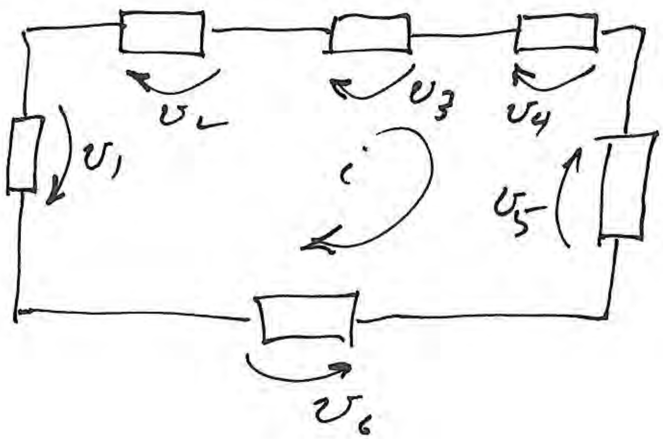
$800 = 2\pi f$
$\Rightarrow f = 800/2\pi$

$SINE(x + 90°) = COSINE(x)$

$SINE(25 + 90) = COSINE(25)$

$SINE(115) = COSINE(25)$

KVL IN THE FREQ DOMAIN



KVL: $v_1 + v_2 \cdots + v_6 = 0$ (TIME DOMAIN)

$$V_{m1}\cos(\omega t + \theta_1) + V_{m2}\cos(\omega t + \theta_2) + \cdots + V_{m6}\cos(\omega t + \theta_6) = 0$$

USING EULER'S IDENTITY:

$$Re\{V_{m1} e^{j(\omega t + \theta_1)}\} + Re\{V_{m2} e^{j(\omega t + \theta_2)}\} + \cdots + Re\{V_{m6} e^{j(\omega t + \theta_6)}\} = 0$$

BECAUSE $Re\{A\} + Re\{B\} = Re\{A + B\}$

WE CAN WRITE

$$\mathrm{Re}\{V_{m1}e^{j(\omega t+\theta_1)}+\cdots+V_{m6}e^{j(\omega t+\theta_6)}\}=0$$

OR

$$\mathrm{Re}\{(V_{m1}e^{j\theta_1}+\cdots+V_{m6}e^{j\theta_6})e^{j\omega t}\}=0$$

THE ONLY WAY TO SATISFY THIS EQ FOR AN ARBITRARY TIME IS TO SET THE TERM

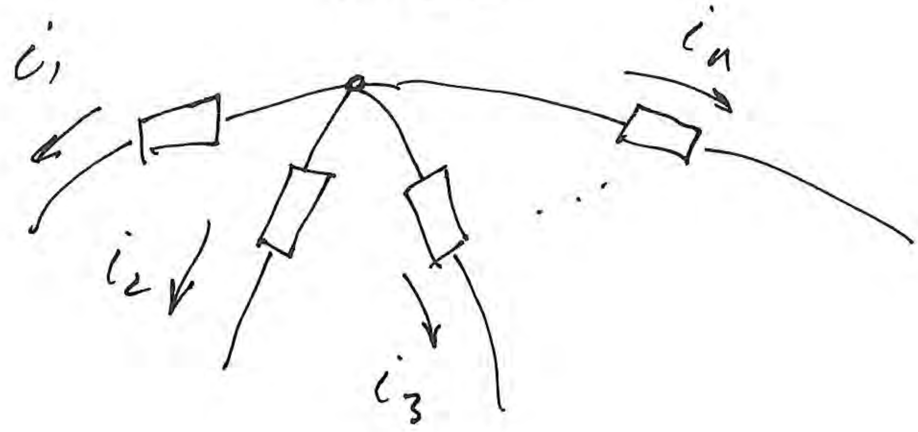
$$(V_{m1}e^{j\theta_1}+\cdots+V_{m6}e^{j\theta_6})=0$$

OR

$$\boxed{\bar{V}_1+\bar{V}_2+\cdots+\bar{V}_6=0} \quad \text{KVL}$$

(FREQUENCY DOMAIN)

KCL

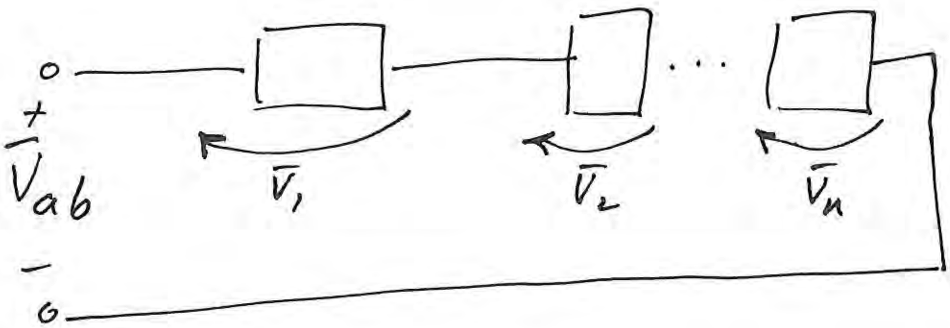


ON BASIS OF PREVIOUS, WE HAVE

$$i_1 + i_2 + \cdots + i_n = 0$$ TIME DOMAIN

$$\bar{I}_1 + \bar{I}_2 + \cdots + \bar{I}_n = 0$$ FREQ DOMAIN

IMPLICATIONS OF KVL & KCL IN THE FREQUENCY DOMAIN



KVL: $\bar{V}_{ab} = \bar{V}_1 + \bar{V}_2 + \cdots + \bar{V}_n$

BUT WE'VE ESTABLISHED THAT REGARDLESS OF COMPONENT, THE $\bar{V}$-$\bar{I}$ RELATIONSHIPS ARE ALL OF THE FORM

$$\bar{V} = Z\bar{I}$$

SIINCE THERE'S ONLY ONE CURRENT IN THIS MESH EQ,

$$\bar{V}_i = Z_i\bar{I}$$

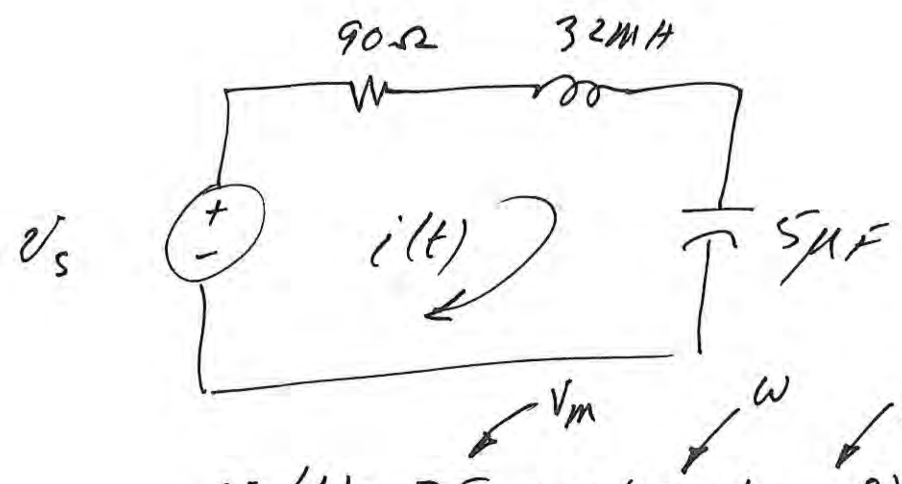
$$\bar{V}_{ab} = z_1\bar{I} + z_2\bar{I} + \cdots + z_n\bar{I}$$

$$= (z_1 + z_2 + \cdots + z_n)\bar{I}$$

GOOD NEWS: SERIES COMBINATIONS COMBINE JUST LIKE RESISTORS

BAD NEWS: WE HAVE TO ADD COMPLEX NUMBERS (SMALL PRICE TO PAY)

EXAMPLE 9.6



V_m ω φ

$$v_s(t) = 750\cos(5{,}000t + 30°)\ \text{V}$$

TIME DOMAIN

$$v_S(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(x)dx$$

FREQUENCY DOMAIN

$$\bar{V}_S = Z_R\bar{I} + Z_L\bar{I} + Z_C\bar{I}$$

$$= (Z_R + Z_L + Z_C)\bar{I}$$

$$\bar{I} = \frac{\bar{V}_S}{Z_{EQ}}$$

$$i(t) = P^{-1}\left\{\frac{\bar{V}_S}{Z_{EQ}}\right\}$$

$$Z_R = 90\,\Omega$$

$$Z_L = j\omega L = j(5{,}000\,RAD/s)(32\,mH)$$

$$= j160\,\Omega$$

$$Z_C = \frac{-j}{\omega C} = -j40\,\Omega$$

$$Z_{EQ} = 90\,\Omega + j160\,\Omega - j40\,\Omega = (90 + j120)\,\Omega$$

$$Z_{EQ} = (90 + j120)\,\Omega$$

$$= \sqrt{(90)^2 + (120)^2}\, e^{j\theta}$$

$$\theta = \tan^{-1}\left(\frac{120}{90}\right) = 53.13^\circ$$

$$Z_{EQ} = 150\, e^{j53.13^\circ}$$

$$Z_{EQ} = 150 \angle 53.13^\circ$$

$$\bar{V}_s = \mathcal{P}\{750 \cos(5{,}000t + 30^\circ)\}$$

$$= 750\, e^{j30^\circ}$$

$$\bar{I} = \frac{\bar{V}_s}{Z_{EQ}} = \frac{750\, e^{j30^\circ}}{150\, e^{j53.13^\circ}}$$

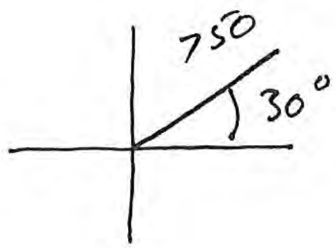
$$= 5\, e^{-j23.13^\circ}$$

$$i(t) = \mathcal{P}^{-1}\{5\, e^{-j23.13^\circ}\}$$

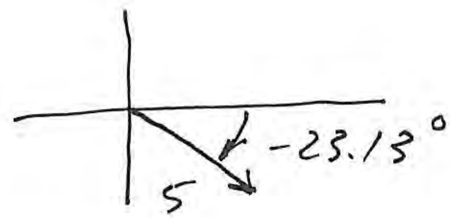
$$i(t) = 5 \cos(5{,}000t - 23.13^\circ)\ A$$

$$v_S(t) = 750\cos(5{,}000t + 30°)\ V$$

$$i(t) = 5\cos(5{,}000t - 23.13°)\ A$$



$v_S(0)$



$i(0)$

$$Z_{EQ} = 150\angle 53.13$$

$$= 150\cos(53.13) + j150\sin(53.13)$$

$$= 90 + j120\ \Omega$$

WHAT IS THE CHARACTER OF THIS IMPEDANCE?

DOES IT HAVE AN INDUCTIVE OR A CAPACITIVE REACTANCE?

$$Z_{EQ} = R + j\omega L - j/\omega C$$

WHAT HAPPENS IF

$$j\omega L - j/\omega C = 0 \quad ?$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

WHERE HAVE WE SEEN THIS FORMULA BEFORE?

WHAT WAS THIS CALLED?

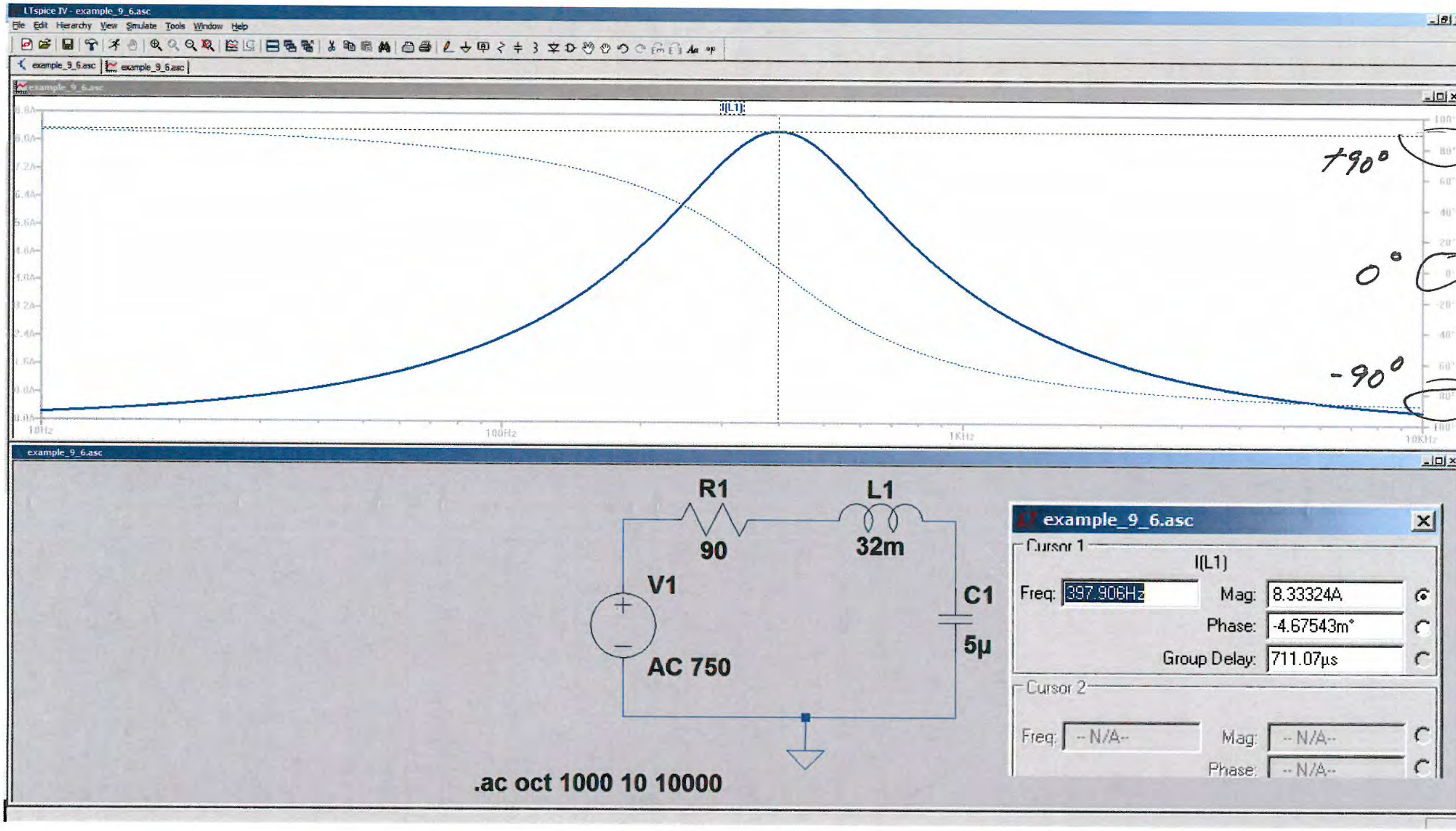
WHY DO YOU SUPPOSE IT HAS THIS NAME?

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(32mH)(5\mu F)}} = 2{,}500 \text{ RAD/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = 397.89 \text{ Hz}$$

$$\text{MAX MESH CURRENT} = \frac{750V}{90\Omega} = 8.33A$$

Example 9.6

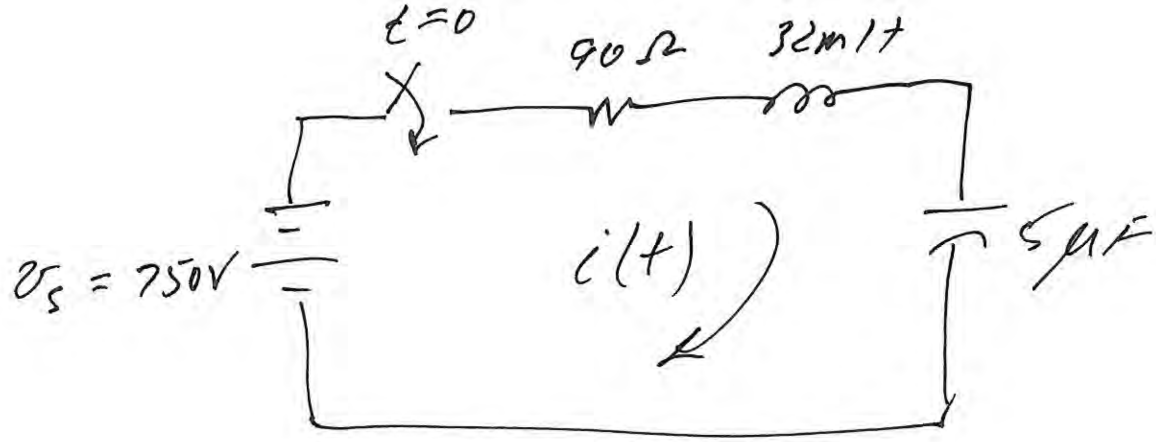


What is illustrated here?
What conclusions can you draw?

L.F. CURRENT LEADS BY 90°

H.F. CURRENT LAGS BY 90°

TRANSIENT RESP



TIME DOMAIN

$$t \geq 0 \qquad v_s = R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(x)\,dx$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{(32mH)(5\mu F)} = 6.25\times10^6\,(RAD/S)^2$$

$$\omega_0 = 2{,}500\ RAD/S$$

$$\alpha = \frac{R}{2L} = \frac{90}{2\,(32mH)} = 1{,}406\ NP/S$$

$$S_{1,2} = -1{,}406 \pm \sqrt{(1{,}406)^2 - 6.25\times10^6}$$

$$= -1{,}406 \pm j\,2{,}067$$

UNDERDAMPED

$$i(t) = B_1 e^{-\alpha t} \cos \omega_0 t + B_2 e^{-\alpha t} \sin \omega_0 t$$

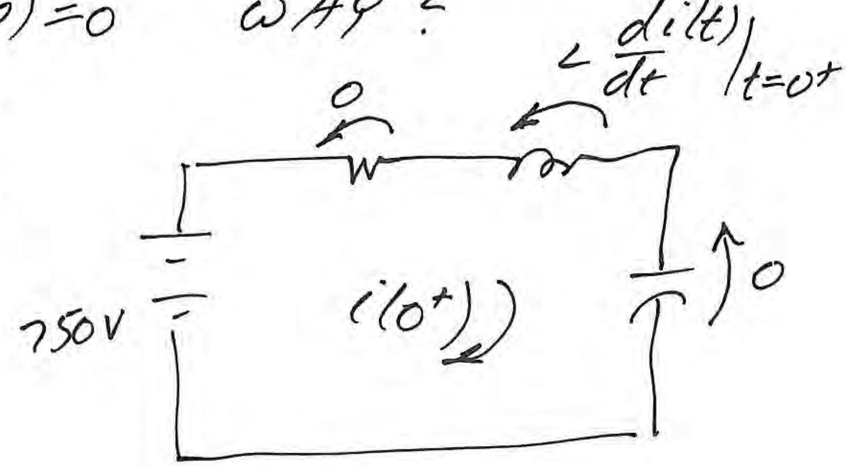
INITIAL COND'S.

ASSUME NO STORED ENERGY

$\Rightarrow i(0) = 0$ WHY?

$\Rightarrow B_1 = 0$

CKT AT $t=0^+$



KVL: $750V = L \frac{di(t)}{dt}\Big|_{t=0^+}$

$$\frac{750V}{32mH} = -\alpha B_1 + \omega_0 B_2$$

$$= \omega_0 B_2$$

$$B_2 = \frac{750V}{32mH(2{,}500)} = 9.375\ A$$

$$i(t) = 9.375\, e^{-1{,}406t} \sin 2{,}500t \text{ A}$$

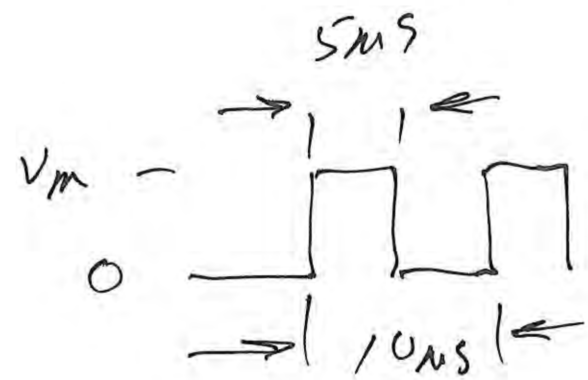
PULSE ANALYSIS

STOP TIME = 20ms

PULSE WIDTH 5ms
PULSE PERIOD = 10ms } 50% DUTY CYCLE

PULSE RISE TIME = 0.1μs

PULSE FALL TIME = 0.1μs



$$i(t) = 9.375e^{-1,406t} \sin 2,500t \quad A$$

$$2,500 = 2\pi f = \frac{2\pi}{T}$$

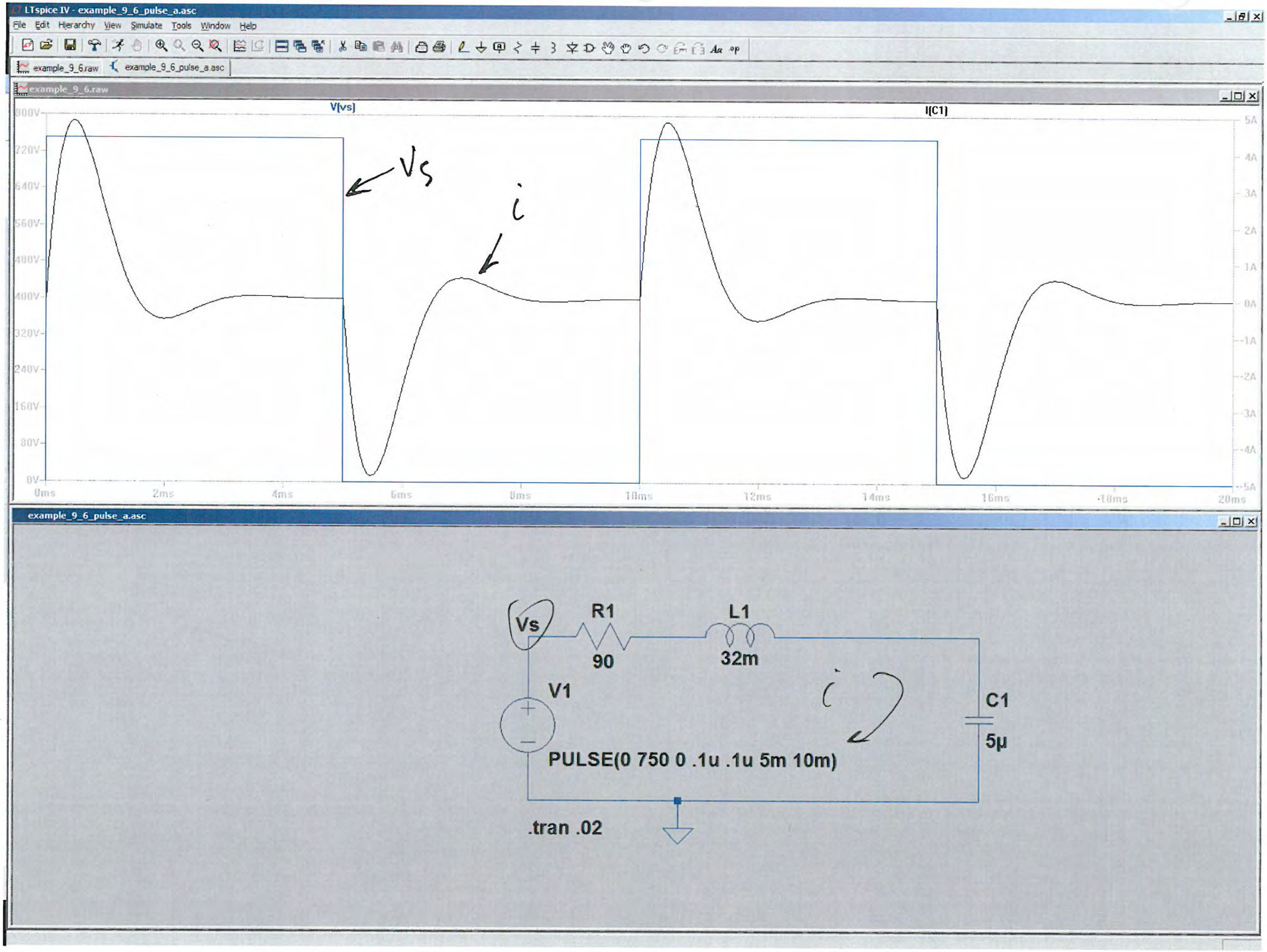
$$T = \frac{2\pi}{2,500} = 2.513 \text{ ms}$$

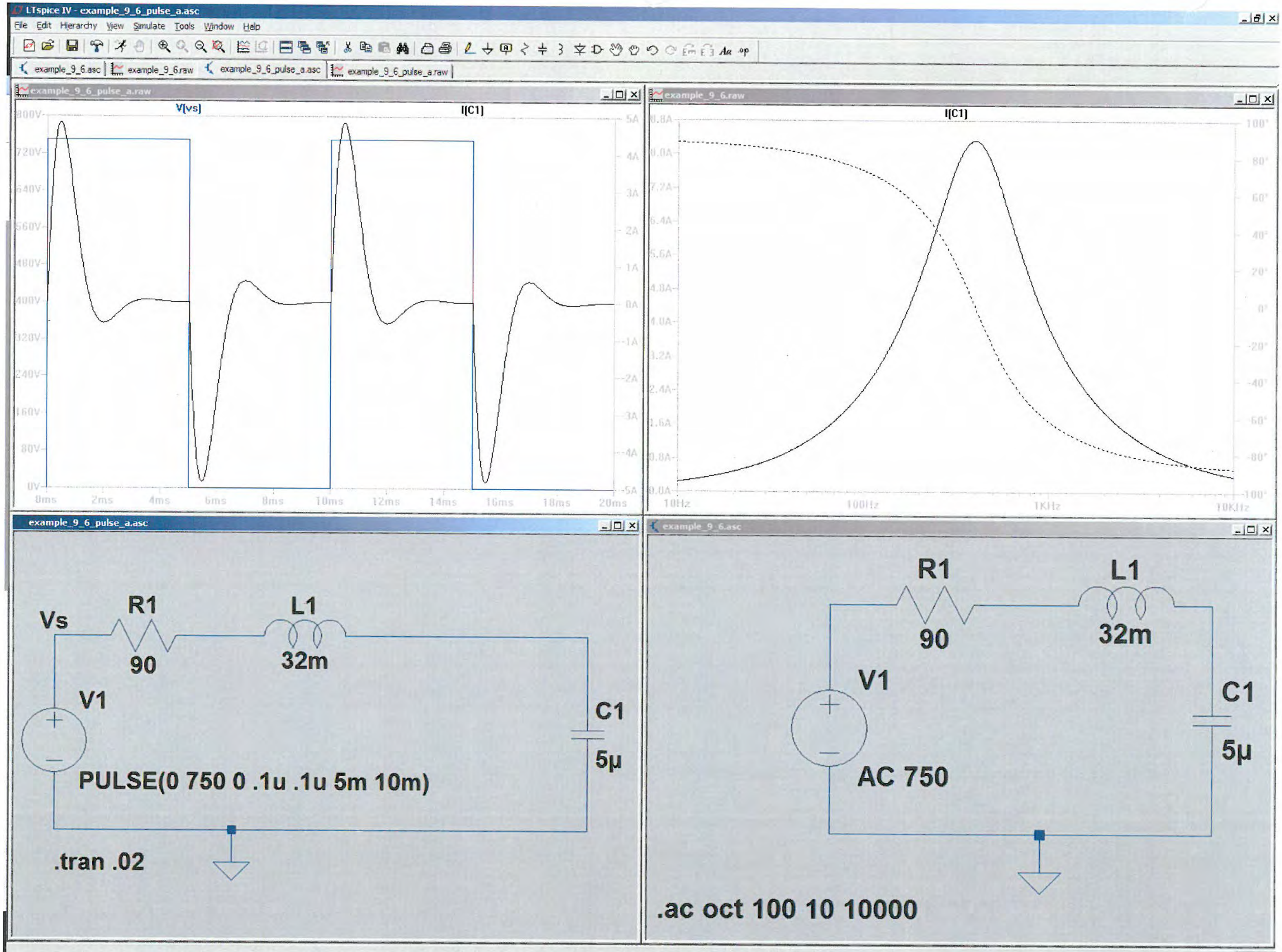
SQ WAVE PERIOD = 10 ms

PULSE WIDTH = 5 ms

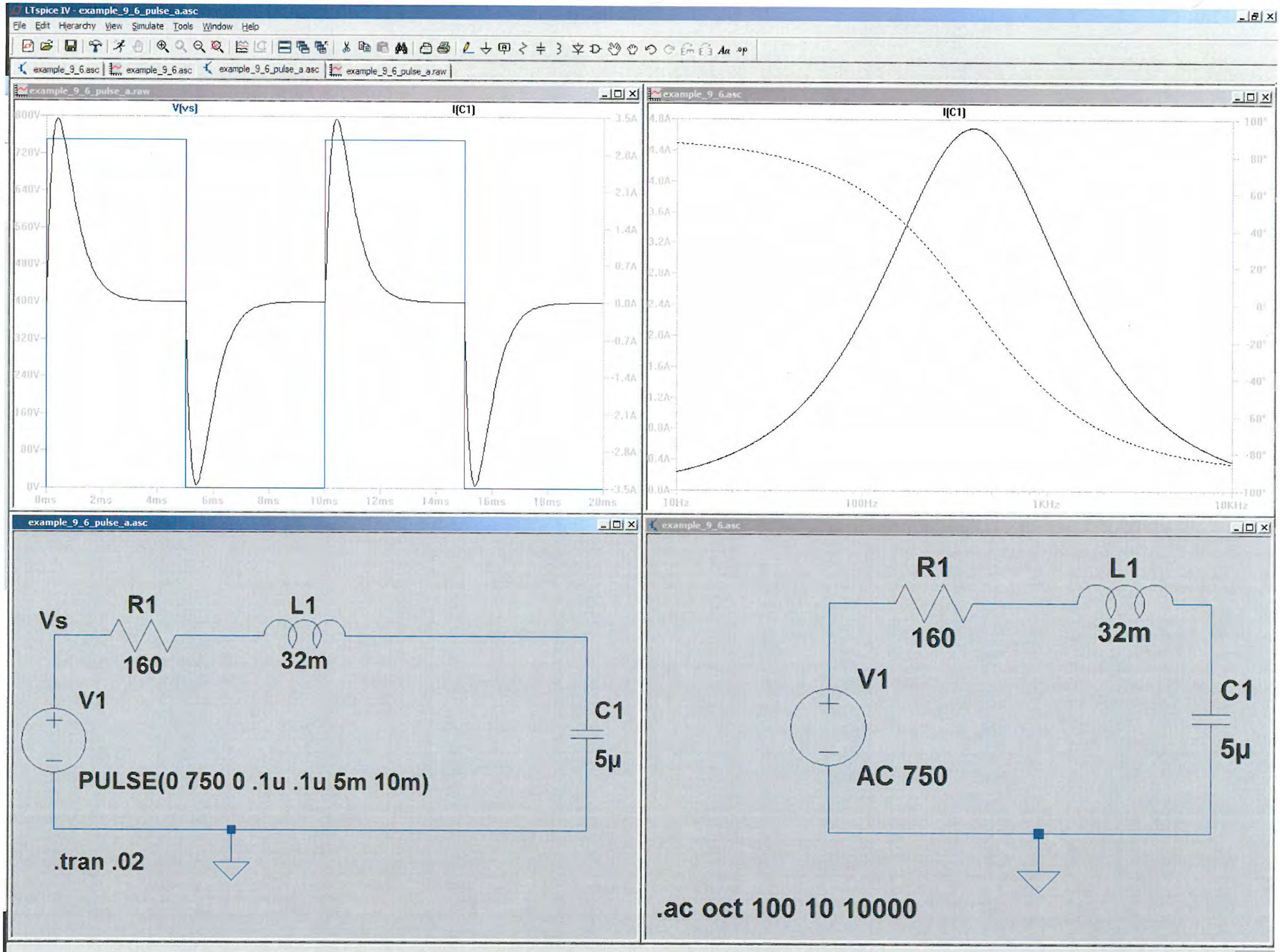
HOW MANY OSCILLATIONS IN 5 ms ?

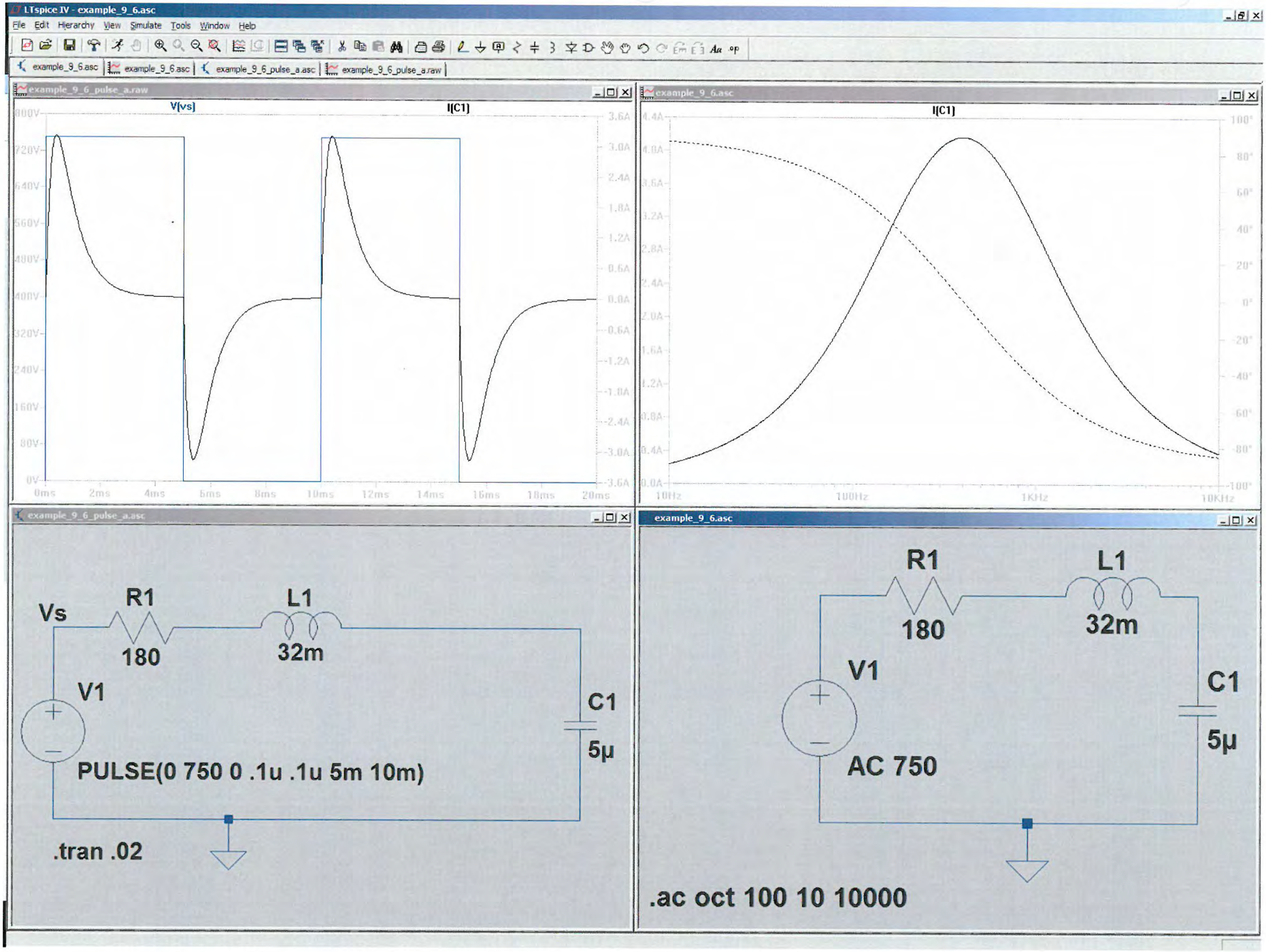
$$\frac{5 \times 10^{-3}}{2.513 \times 10^{-3}} \approx \underline{\underline{2}}$$


LTspice IV - example_9_6_pulse_a.asc
File Edit Hierarchy View Simulate Tools Window Help
example_9_6.raw
example_9_6_pulse_a.asc
V(vs)
I(C1)
800V 720V 640V 560V 480V 400V 320V 240V 160V 80V 0V
5A 4A 3A 2A 1A 0A -1A -2A -3A -4A -5A
0ms 2ms 4ms 6ms 8ms 10ms 12ms 14ms 16ms 18ms 20ms
Vs
i
R1
90
L1
32m
V1
C1
5µ
PULSE(0 750 0 .1u .1u 5m 10m)
.tran .02



LTspice IV - example_9_6_pulse_a.asc
File Edit Hierarchy View Simulate Tools Window Help
example_9_6.asc
example_9_6.raw
example_9_6_pulse_a.asc
example_9_6_pulse_a.raw
example_9_6_pulse_a.raw
V(vs)
I(C1)
800V
720V
640V
560V
480V
400V
320V
240V
160V
80V
0V
5A
4A
3A
2A
1A
0A
-1A
-2A
-3A
-4A
-5A
0ms
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
20ms
example_9_6.raw
I(C1)
8.8A
8.0A
7.2A
6.4A
5.6A
4.8A
4.0A
3.2A
2.4A
1.6A
0.8A
0.0A
100°
80°
60°
40°
20°
0°
-20°
-40°
-60°
-80°
-100°
10Hz
100Hz
1KHz
10KHz
example_9_6_pulse_a.asc
Vs
R1
90
L1
32m
V1
C1
5μ
PULSE(0 750 0 .1u .1u 5m 10m)
.tran .02
example_9_6.asc
R1
90
L1
32m
V1
C1
5μ
AC 750
.ac oct 100 10 10000


LTspice IV - example_9_6_pulse_a.asc
File Edit Hierarchy View Simulate Tools Window Help
example_9_6.asc
example_9_6.asc
example_9_6_pulse_a.asc
example_9_6_pulse_a.raw
example_9_6_pulse_a.raw
V(vs)
I(C1)
800V
720V
640V
560V
480V
400V
320V
240V
160V
80V
0V
3.5A
2.8A
2.1A
1.4A
0.7A
0.0A
-0.7A
-1.4A
-2.1A
-2.8A
-3.5A
0ms 2ms 4ms 6ms 8ms 10ms 12ms 14ms 16ms 18ms 20ms
example_9_6.asc
I(C1)
4.8A
4.4A
4.0A
3.6A
3.2A
2.8A
2.4A
2.0A
1.6A
1.2A
0.8A
0.4A
0.0A
100°
80°
60°
40°
20°
0°
-20°
-40°
-60°
-80°
-100°
10Hz 100Hz 1KHz 10KHz
example_9_6_pulse_a.asc
Vs
R1
160
L1
32m
V1
C1
5μ
PULSE(0 750 0 .1u .1u 5m 10m)
.tran .02
example_9_6.asc
R1
160
L1
32m
V1
C1
5μ
AC 750
.ac oct 100 10 10000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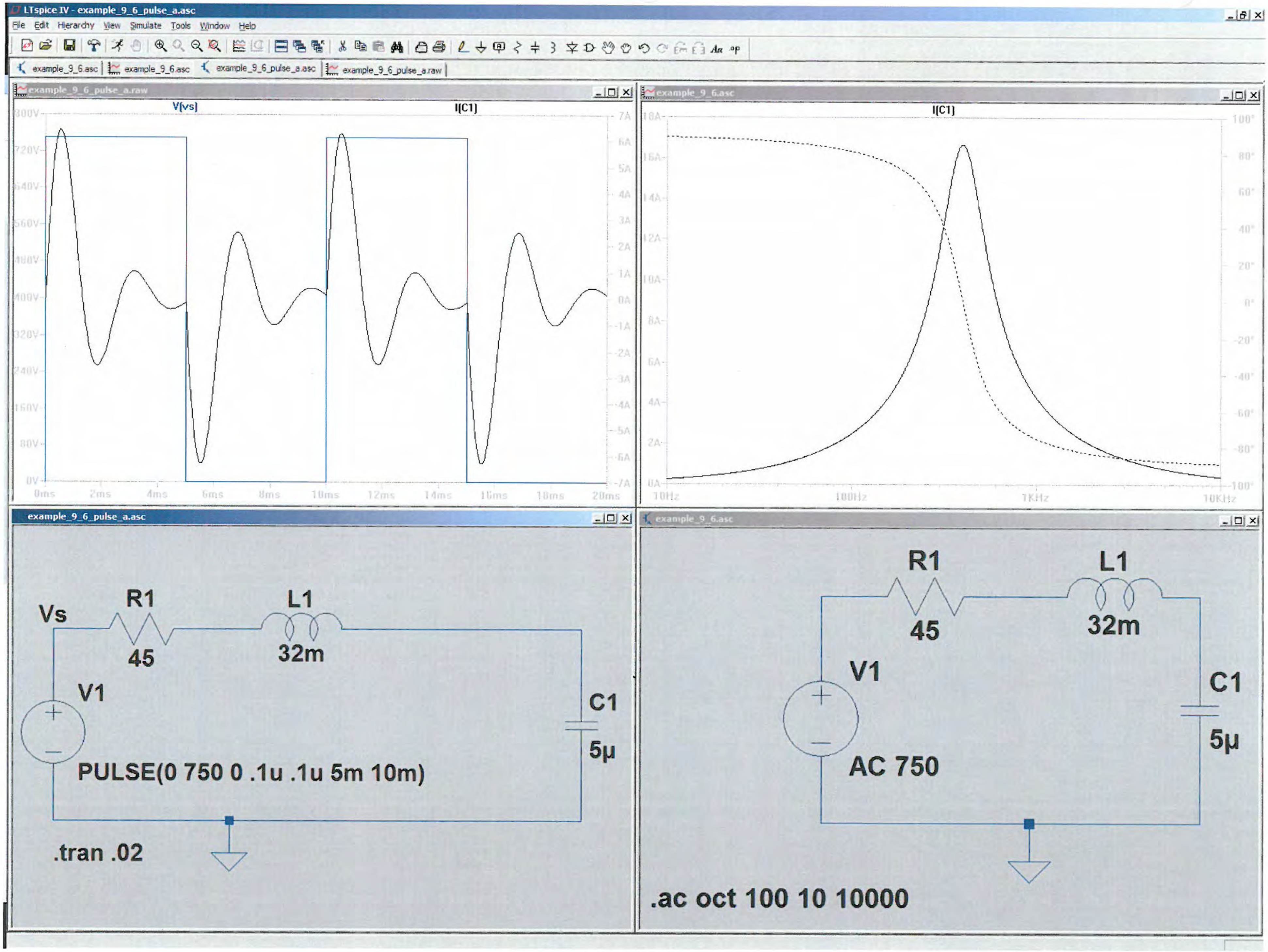
example_9_6.asc
example_9_6.asc
example_9_6_pulse_a.asc
example_9_6_pulse_a.raw
example_9_6_pulse_a.raw
V(vs)
I(C1)
800V
720V
640V
560V
480V
400V
320V
240V
160V
80V
0V
3.6A
3.0A
2.4A
1.8A
1.2A
0.6A
0.0A
-0.6A
-1.2A
-1.8A
-2.4A
-3.0A
-3.6A
0ms
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
20ms
example_9_6.asc
I(C1)
4.4A
4.0A
3.6A
3.2A
2.8A
2.4A
2.0A
1.6A
1.2A
0.8A
0.4A
0.0A
100°
80°
60°
40°
20°
0°
-20°
-40°
-60°
-80°
-100°
10Hz
100Hz
1KHz
10KHz
example_9_6_pulse_a.asc
Vs
R1
180
L1
32m
V1
C1
5µ
PULSE(0 750 0 .1u .1u 5m 10m)
.tran .02
example_9_6.asc
R1
180
L1
32m
V1
C1
5µ
AC 750
.ac oct 100 10 10000



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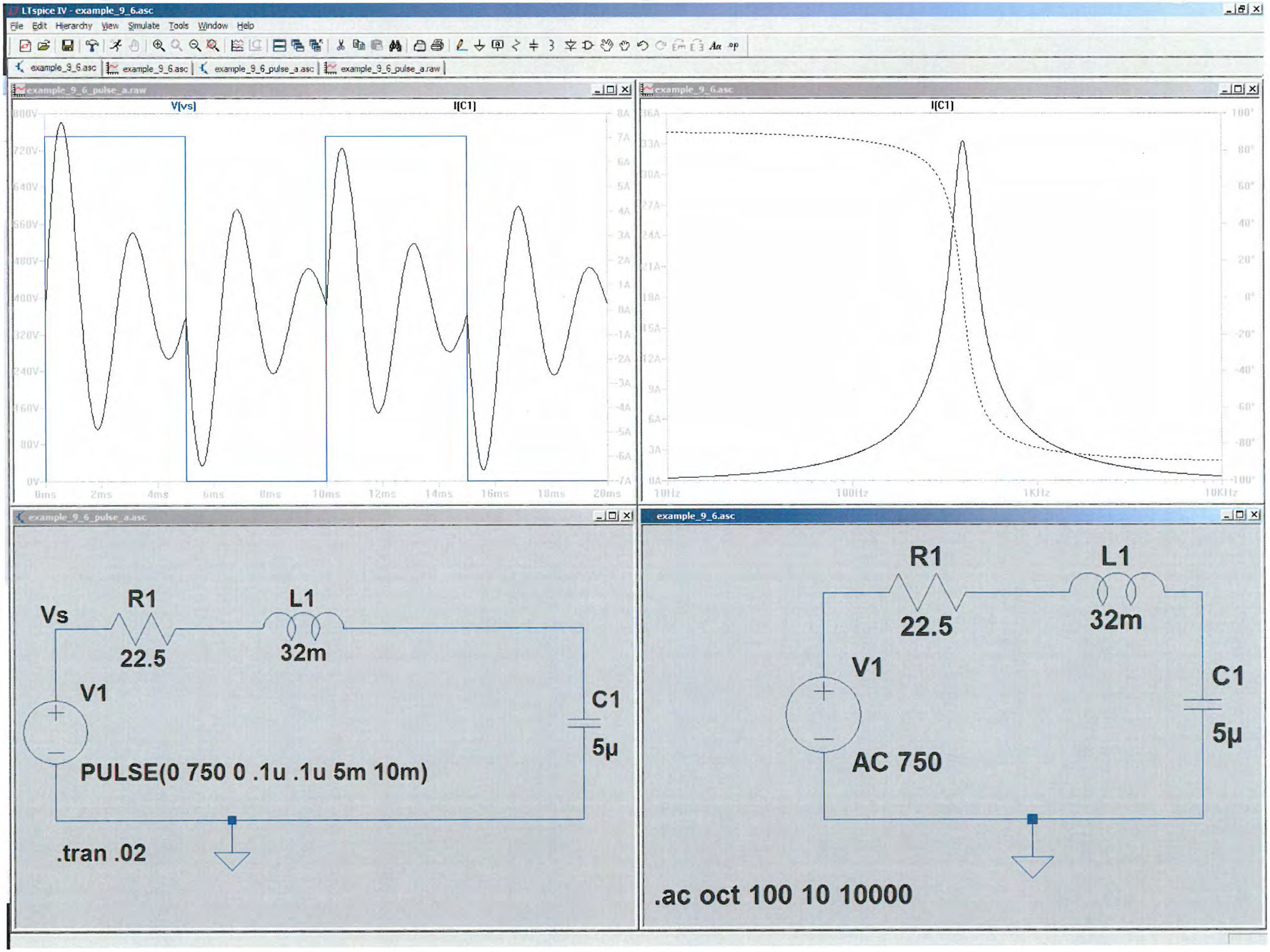
File Edit Hierarchy View Simulate Tools Window Help
example_9_6.asc
example_9_6.asc
example_9_6_pulse_a.asc
example_9_6_pulse_a.raw
example_9_6_pulse_a.raw
V(vs)
I(C1)
800V 720V 640V 560V 480V 400V 320V 240V 160V 80V 0V
7A 6A 5A 4A 3A 2A 1A 0A -1A -2A -3A -4A -5A -6A -7A
0ms 2ms 4ms 6ms 8ms 10ms 12ms 14ms 16ms 18ms 20ms
example_9_6.asc
I(C1)
18A 16A 14A 12A 10A 8A 6A 4A 2A 0A
100° 80° 60° 40° 20° 0° -20° -40° -60° -80° -100°
10Hz 100Hz 1KHz 10KHz
example_9_6_pulse_a.asc
Vs
R1
45
L1
32m
V1
C1
5µ
PULSE(0 750 0 .1u .1u 5m 10m)
.tran .02
example_9_6.asc
R1
45
L1
32m
V1
C1
5µ
AC 750
.ac oct 100 10 10000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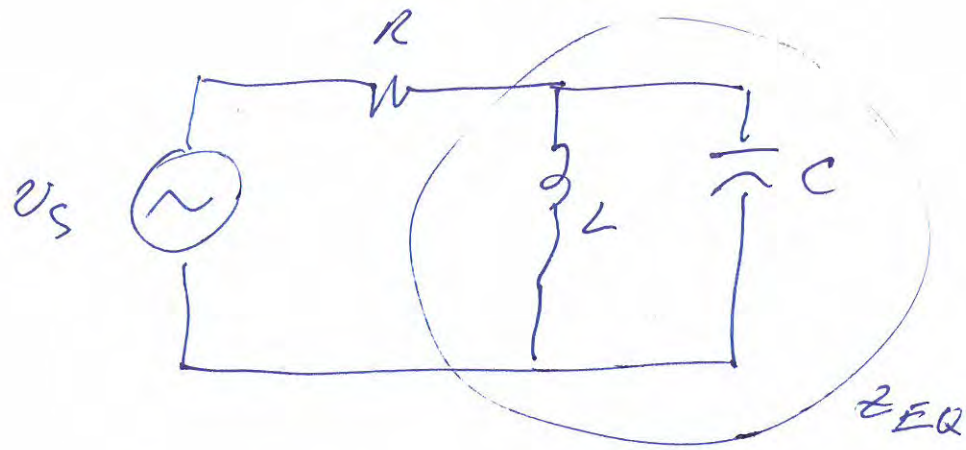
example_9_6.asc
example_9_6.asc
example_9_6_pulse_a.asc
example_9_6_pulse_a.raw
example_9_6_pulse_a.raw
V(vs)
I(C1)
800V
720V
640V
560V
480V
400V
320V
240V
160V
80V
0V
8A
7A
6A
5A
4A
3A
2A
1A
0A
-1A
-2A
-3A
-4A
-5A
-6A
-7A
0ms
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
20ms
example_9_6.asc
I(C1)
36A
33A
30A
27A
24A
21A
18A
15A
12A
9A
6A
3A
0A
100°
80°
60°
40°
20°
0°
-20°
-40°
-60°
-80°
-100°
10Hz
100Hz
1KHz
10KHz
example_9_6_pulse_a.asc
Vs
R1
22.5
L1
32m
V1
C1
5μ
PULSE(0 750 0 .1u .1u 5m 10m)
.tran .02
example_9_6.asc
R1
22.5
L1
32m
V1
C1
5μ
AC 750
.ac oct 100 10 10000

EXAMPLE



$$v_s(t) = 2\cos(\omega t)\ \text{V}$$

$$z_{EQ} = \frac{j\omega L / j\omega C}{j\omega L + 1/j\omega C} = \frac{j\omega L}{-\omega^2 LC + 1}$$

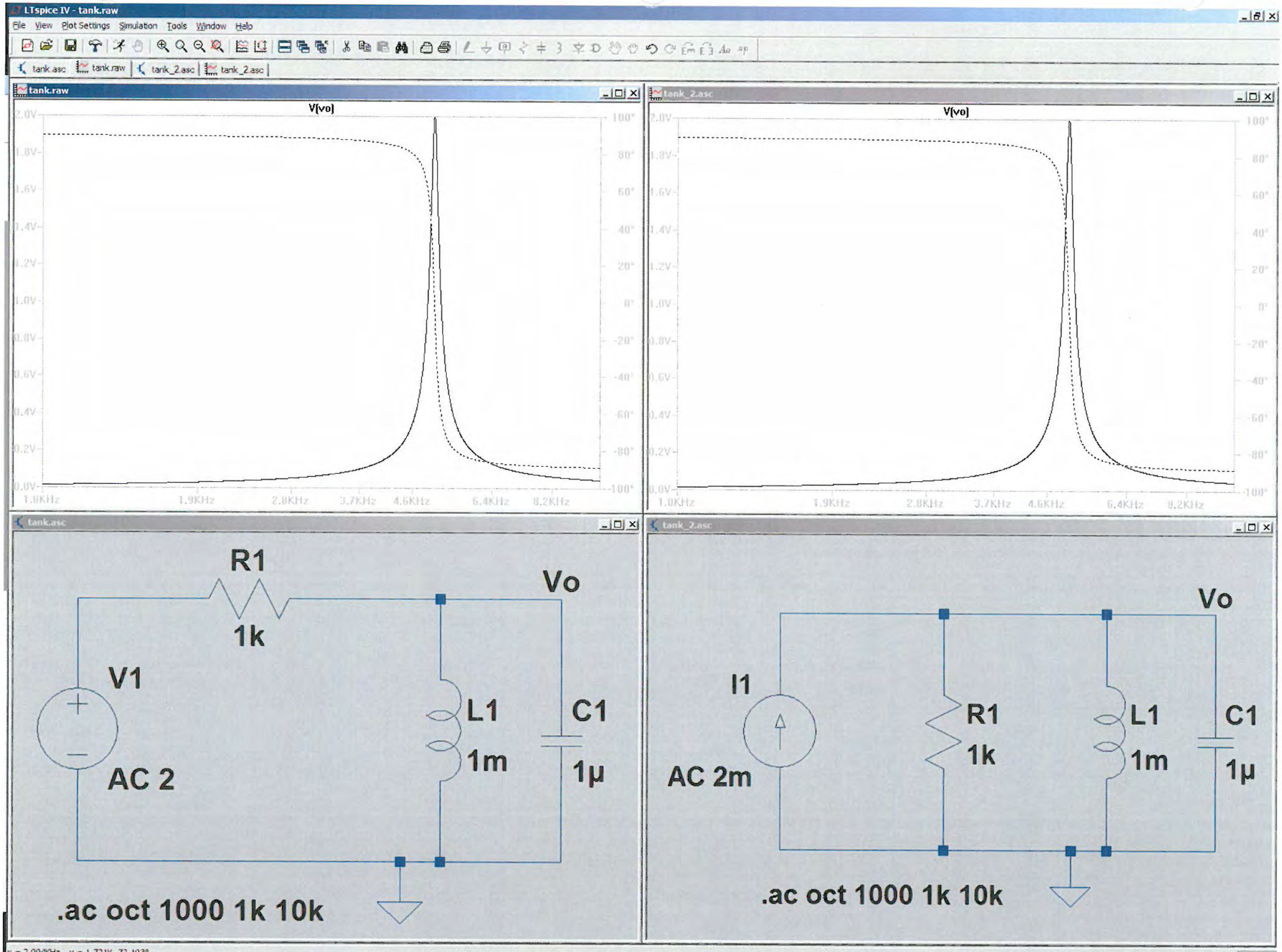
$$LC = 1/\omega_0^2 \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

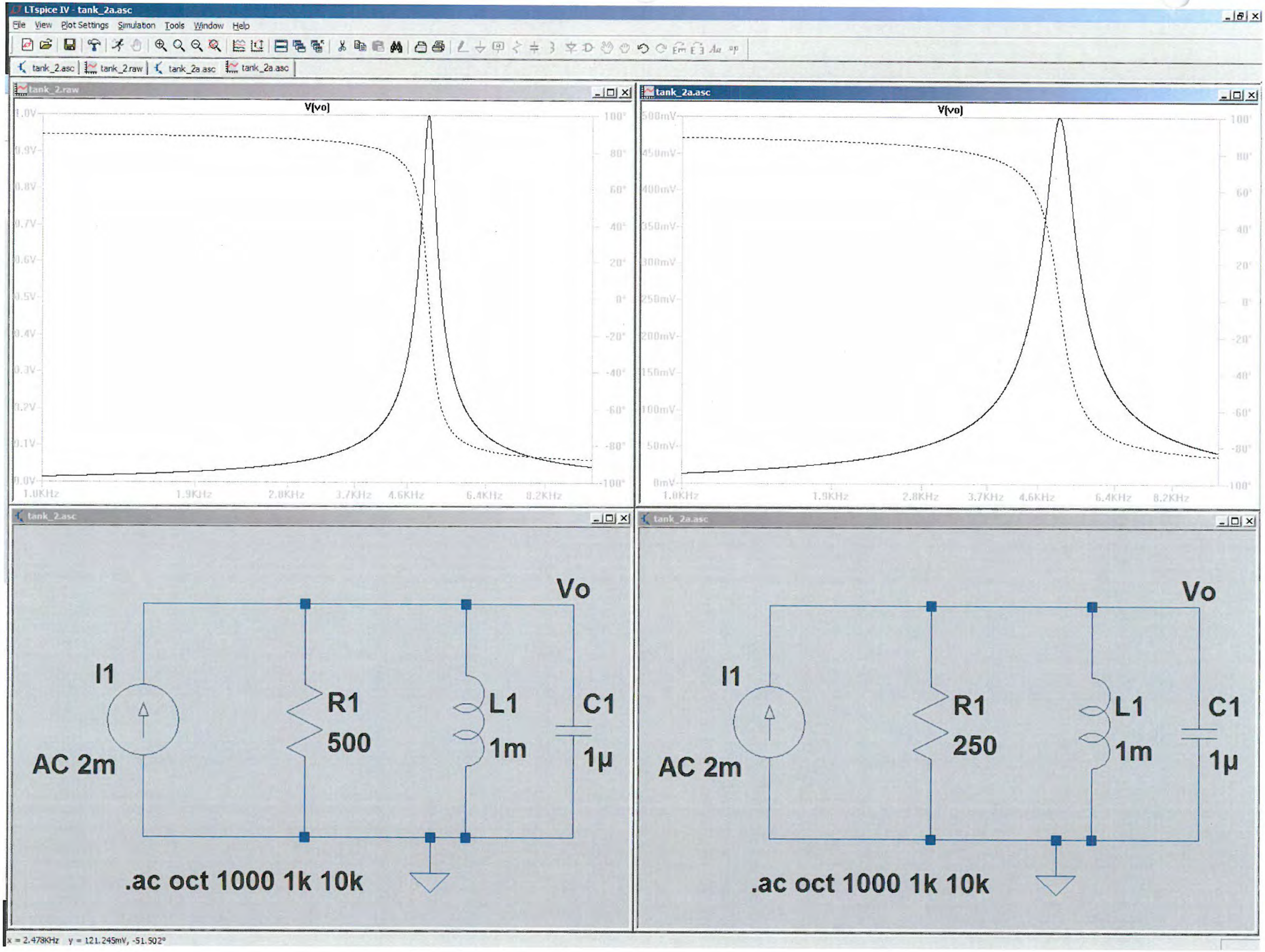
$$z_{EQ} = \frac{j\omega L}{1 - (\omega/\omega_0)^2}$$

$\omega < \omega_0$ REACTANCE POSITIVE

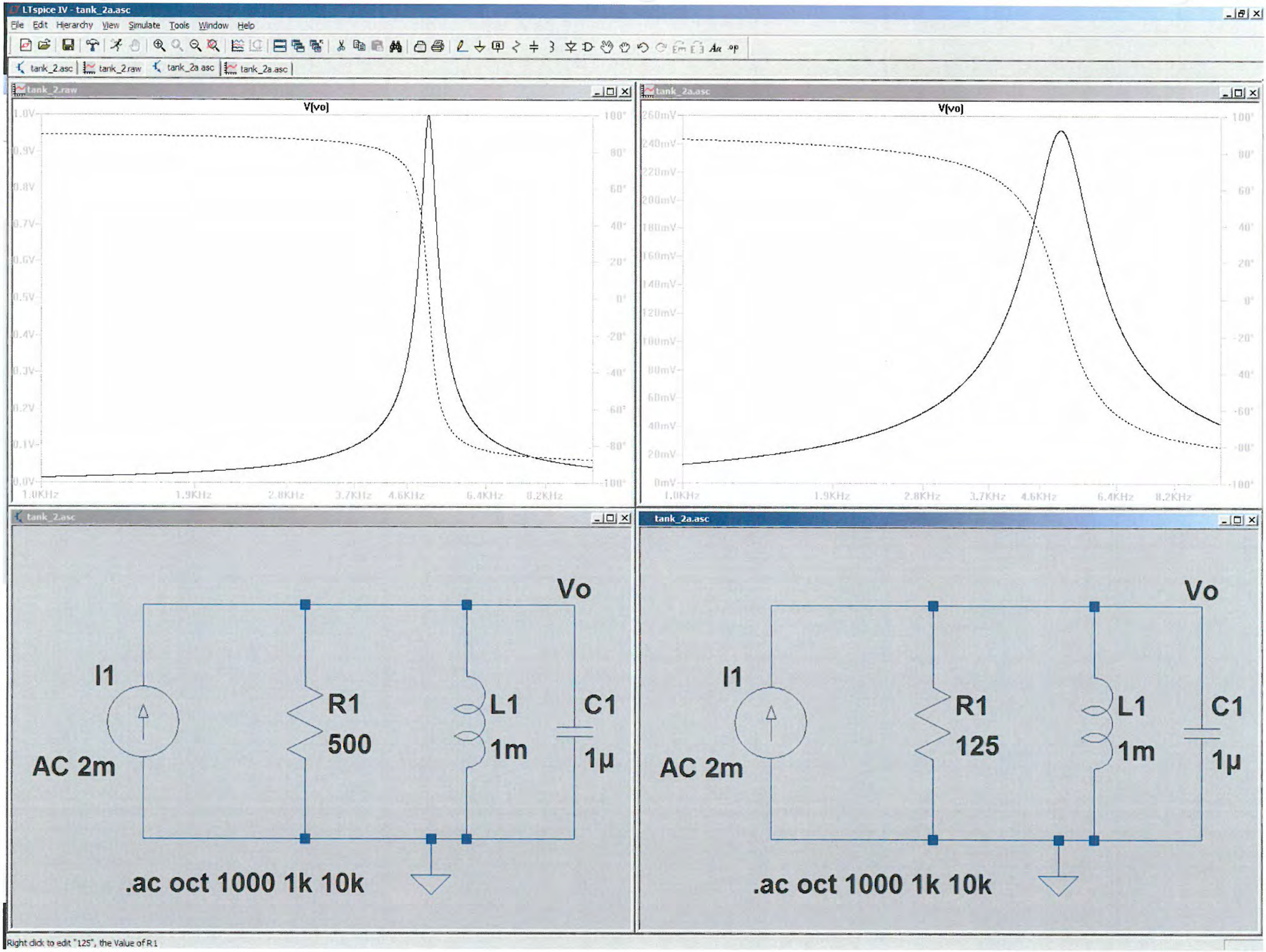
$\omega = \omega_0$ $z_{EQ} \to \infty$

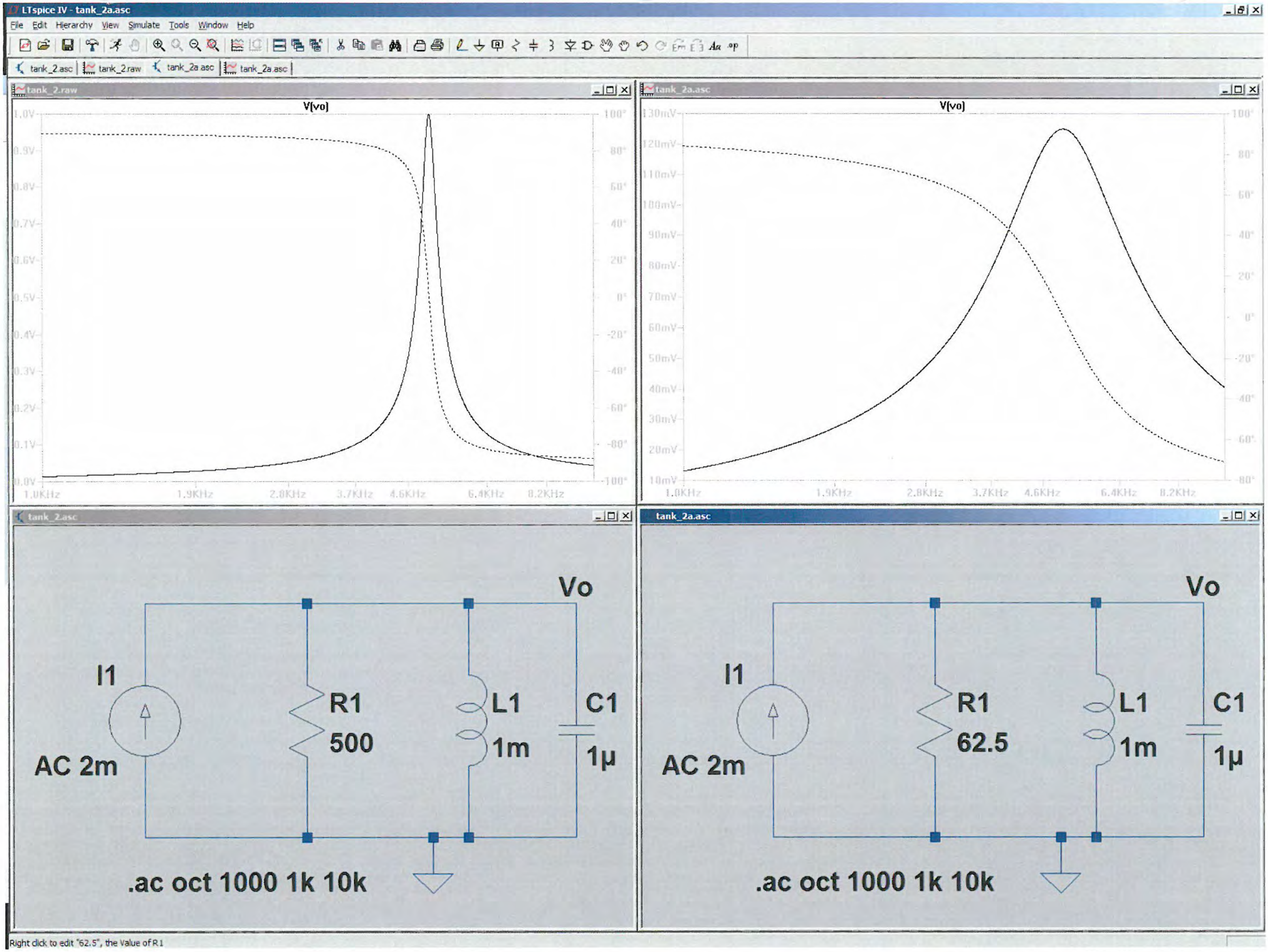
$\omega > \omega_0$ REACTANCE NEGATIVE


LTspice IV - tank.raw
File View Plot Settings Simulation Tools Window Help
tank.asc tank.raw tank_2.asc tank_2.asc
tank.raw
V(vo)
2.0V 1.8V 1.6V 1.4V 1.2V 1.0V 0.8V 0.6V 0.4V 0.2V 0.0V
100° 80° 60° 40° 20° 0° -20° -40° -60° -80° -100°
1.0KHz 1.9KHz 2.8KHz 3.7KHz 4.6KHz 6.4KHz 8.2KHz
tank_2.asc
V(vo)
2.0V 1.8V 1.6V 1.4V 1.2V 1.0V 0.8V 0.6V 0.4V 0.2V 0.0V
100° 80° 60° 40° 20° 0° -20° -40° -60° -80° -100°
1.0KHz 1.9KHz 2.8KHz 3.7KHz 4.6KHz 6.4KHz 8.2KHz
tank.asc
R1
1k
Vo
V1
AC 2
L1
1m
C1
1µ
.ac oct 1000 1k 10k
tank_2.asc
Vo
I1
AC 2m
R1
1k
L1
1m
C1
1µ
.ac oct 1000 1k 10k
x = 2.094KHz y = 1.721V, 72.103°



LTspice IV - tank_2a.asc
File View Plot Settings Simulation Tools Window Help
tank_2.asc tank_2.raw tank_2a.asc tank_2a.asc
tank_2.raw
V(vo)
1.0V 0.9V 0.8V 0.7V 0.6V 0.5V 0.4V 0.3V 0.2V 0.1V 0.0V
100° 80° 60° 40° 20° 0° -20° -40° -60° -80° -100°
1.0KHz 1.9KHz 2.8KHz 3.7KHz 4.6KHz 6.4KHz 8.2KHz
tank_2a.asc
V(vo)
500mV 450mV 400mV 350mV 300mV 250mV 200mV 150mV 100mV 50mV 0mV
100° 80° 60° 40° 20° 0° -20° -40° -60° -80° -100°
1.0KHz 1.9KHz 2.8KHz 3.7KHz 4.6KHz 6.4KHz 8.2KHz
tank_2.asc
Vo
I1
AC 2m
R1
500
L1
1m
C1
1μ
.ac oct 1000 1k 10k
tank_2a.asc
Vo
I1
AC 2m
R1
250
L1
1m
C1
1μ
.ac oct 1000 1k 10k
x = 2.478KHz y = 121.245mV, -51.502°


LTspice IV - tank_2a.asc
File Edit Hierarchy View Simulate Tools Window Help
tank_2.asc
tank_2.raw
tank_2a asc
tank_2a.asc
tank_2.raw
V(vo)
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
100°
80°
60°
40°
20°
0°
-20°
-40°
-60°
-80°
-100°
1.0KHz
1.9KHz
2.8KHz
3.7KHz
4.6KHz
6.4KHz
8.2KHz
tank_2a.asc
V(vo)
260mV
240mV
220mV
200mV
180mV
160mV
140mV
120mV
100mV
80mV
60mV
40mV
20mV
0mV
100°
80°
60°
40°
20°
0°
-20°
-40°
-60°
-80°
-100°
1.0KHz
1.9KHz
2.8KHz
3.7KHz
4.6KHz
6.4KHz
8.2KHz
tank_2.asc
Vo
I1
AC 2m
R1
500
L1
1m
C1
1µ
.ac oct 1000 1k 10k
tank_2a.asc
Vo
I1
AC 2m
R1
125
L1
1m
C1
1µ
.ac oct 1000 1k 10k
Right click to edit "125", the Value of R1


LTspice IV - tank_2a.asc
File Edit Hierarchy View Simulate Tools Window Help
tank_2.asc
tank_2.raw
tank_2a.asc
tank_2a.asc
tank_2.raw
V(vo)
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
100°
80°
60°
40°
20°
0°
-20°
-40°
-60°
-80°
-100°
1.0KHz
1.9KHz
2.8KHz
3.7KHz
4.6KHz
6.4KHz
8.2KHz
tank_2a.asc
V(vo)
130mV
120mV
110mV
100mV
90mV
80mV
70mV
60mV
50mV
40mV
30mV
20mV
10mV
100°
80°
60°
40°
20°
0°
-20°
-40°
-60°
-80°
1.0KHz
1.9KHz
2.8KHz
3.7KHz
4.6KHz
6.4KHz
8.2KHz
tank_2.asc
Vo
I1
AC 2m
R1
500
L1
1m
C1
1μ
.ac oct 1000 1k 10k
tank_2a.asc
Vo
I1
AC 2m
R1
62.5
L1
1m
C1
1μ
.ac oct 1000 1k 10k
Right click to edit "62.5", the Value of R1

SERIES RLC ($\alpha = \frac{R}{2L}$)

R ↑ α ↑ Q ↓ RINGING ↓

PARALLEL RLC ($\alpha = \frac{1}{2RC}$)

R ↑ α ↓ Q ↑ RINGING ↑

CRITICAL DAMPING

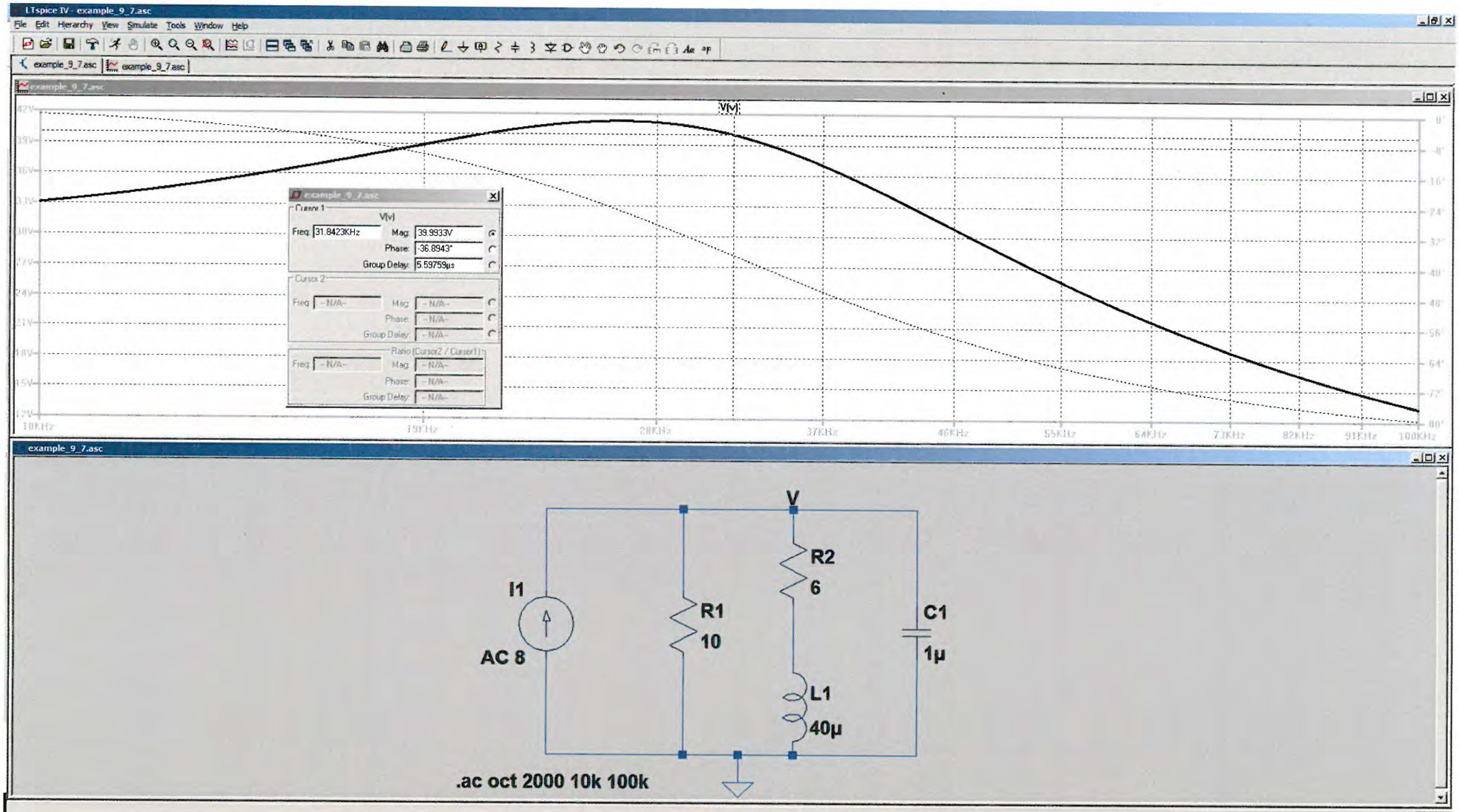
SERIES RLC: $\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\alpha = \omega_0 \Rightarrow R = \frac{2L}{\sqrt{LC}}, \quad \boxed{R = 2\sqrt{\frac{L}{C}}}$$

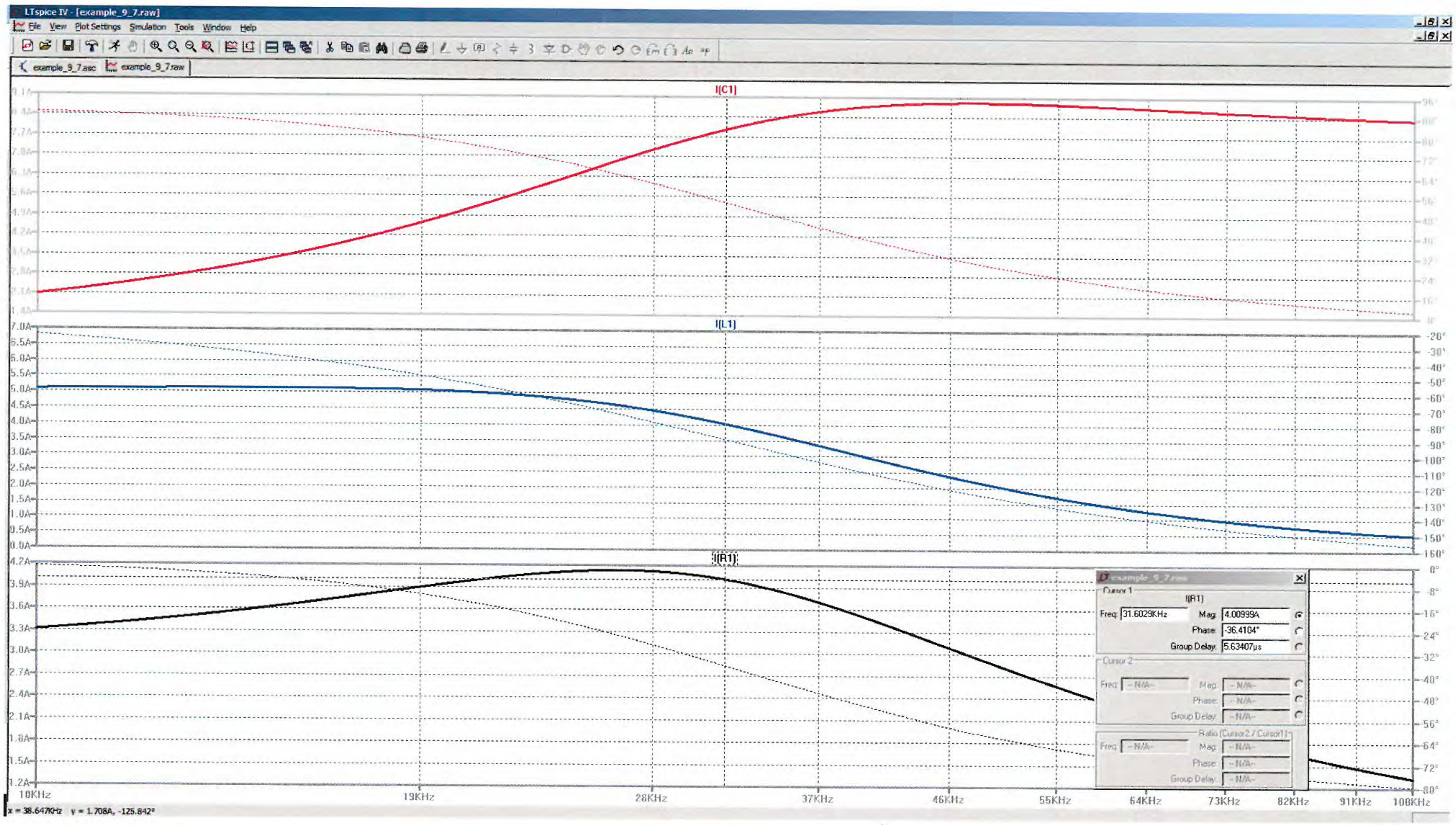
PARALLEL RLC: $\alpha = \frac{1}{2RC}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\alpha = \omega_0 \Rightarrow \frac{\sqrt{LC}}{2C} = R, \quad \boxed{R = \frac{1}{2}\sqrt{\frac{L}{C}}}$$

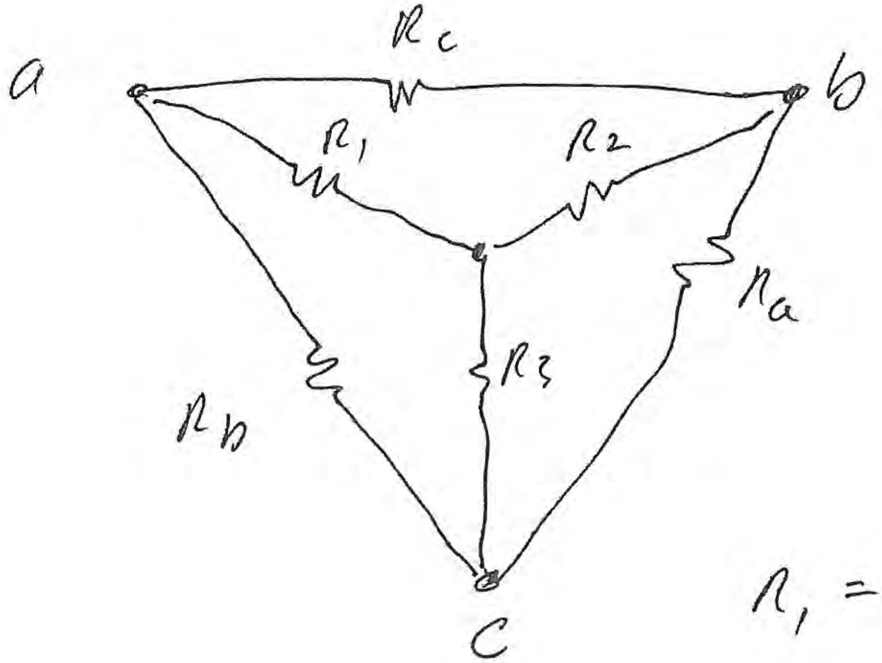
Example 9.7



LTspice IV - example_9_7.asc
File Edit Hierarchy View Simulate Tools Window Help
example_9_7.asc
V(v)
Cursor 1
Freq: 31.8423KHz
Mag: 39.9933V
Phase: -36.8943°
Group Delay: 5.59759µs
Cursor 2
Ratio (Cursor2 / Cursor1)
10KHz
19KHz
28KHz
37KHz
46KHz
55KHz
64KHz
73KHz
82KHz
91KHz
100KHz
V
I1
AC 8
R1
10
R2
6
L1
40µ
C1
1µ
.ac oct 2000 10k 100k


LTspice IV - [example_9_7.raw]
File View Plot Settings Simulation Tools Window Help
example_9_7.asc
example_9_7.raw
I(C1)
I(L1)
I(R1)
example_9_7.raw
Cursor 1
I(R1)
Freq: 31.6029KHz
Mag: 4.00999A
Phase: -36.4104°
Group Delay: 5.63407µs
Cursor 2
Freq: --N/A--
Mag: --N/A--
Phase: --N/A--
Group Delay: --N/A--
Ratio (Cursor2 / Cursor1)
Freq: --N/A--
Mag: --N/A--
Phase: --N/A--
Group Delay: --N/A--
7.0A 6.5A 6.0A 5.5A 5.0A 4.5A 4.0A 3.5A 3.0A 2.5A 2.0A 1.5A 1.0A 0.5A 0.0A
-20° -30° -40° -50° -60° -70° -80° -90° -100° -110° -120° -130° -140° -150° -160°
4.2A 3.9A 3.6A 3.3A 3.0A 2.7A 2.4A 2.1A 1.8A 1.5A 1.2A
0° -8° -16° -24° -32° -40° -48° -56° -64° -72° -80°
10KHz 19KHz 28KHz 37KHz 46KHz 55KHz 64KHz 73KHz 82KHz 91KHz 100KHz
x = 38.647KHz y = 1.708A, -125.842°

$\Delta \leftrightarrow Y$



$$R_1 = \frac{R_b R_c}{R_T}$$

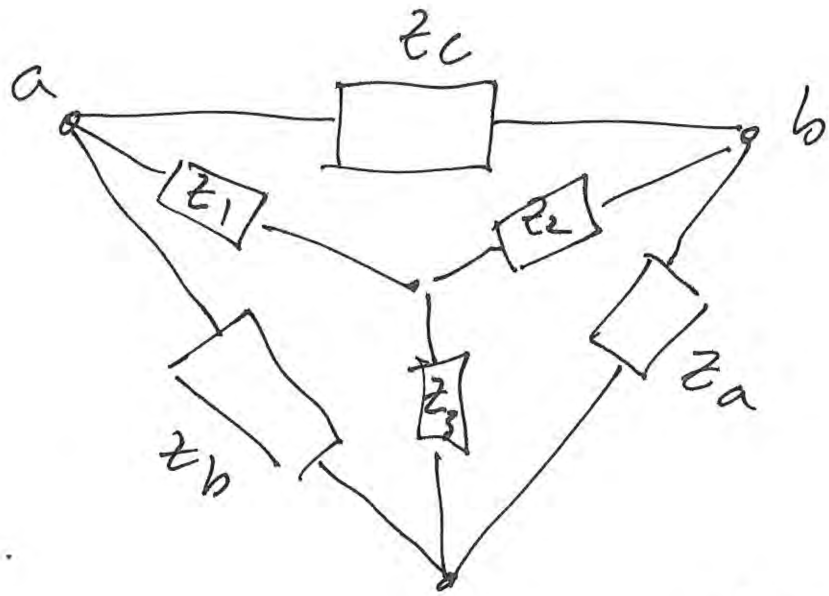
$$R_2 = \frac{R_a R_c}{R_T}$$

$$R_3 = \frac{R_a R_b}{R_T}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$



$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

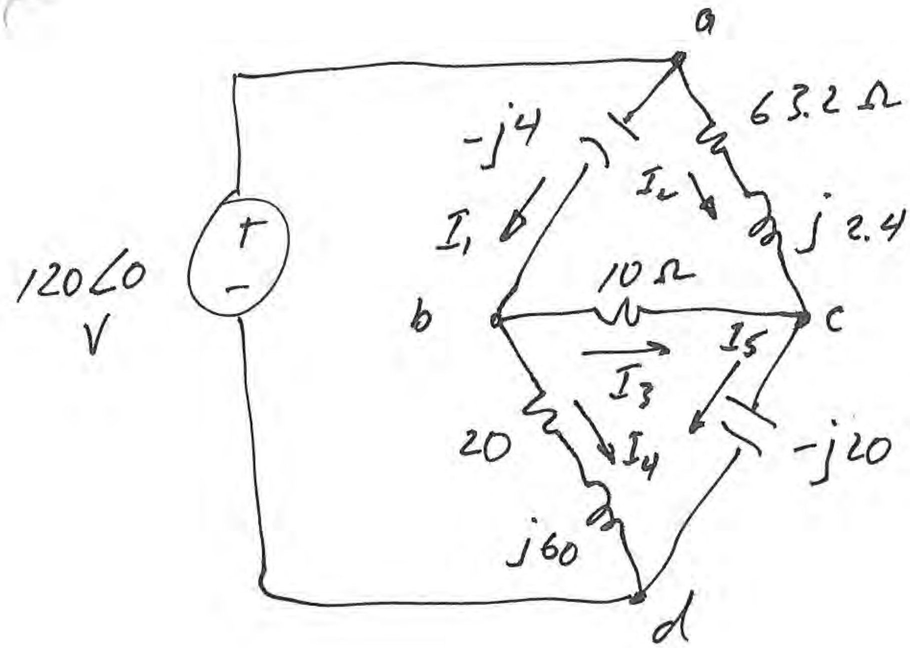
$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

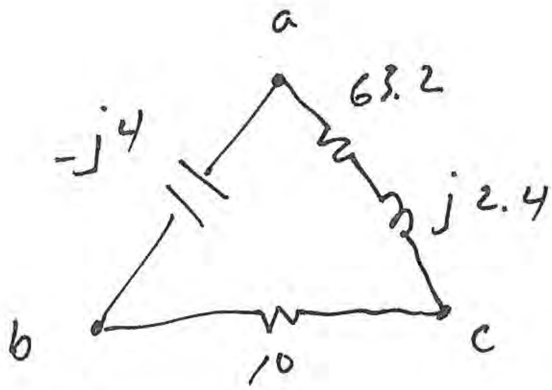
$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

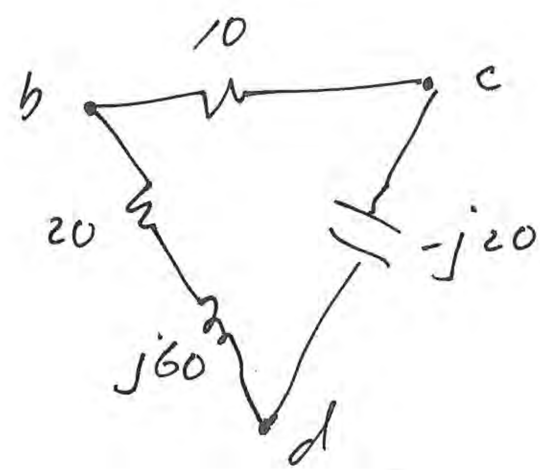
$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

EXAMPLE 9.8 $\Delta \rightarrow Y$







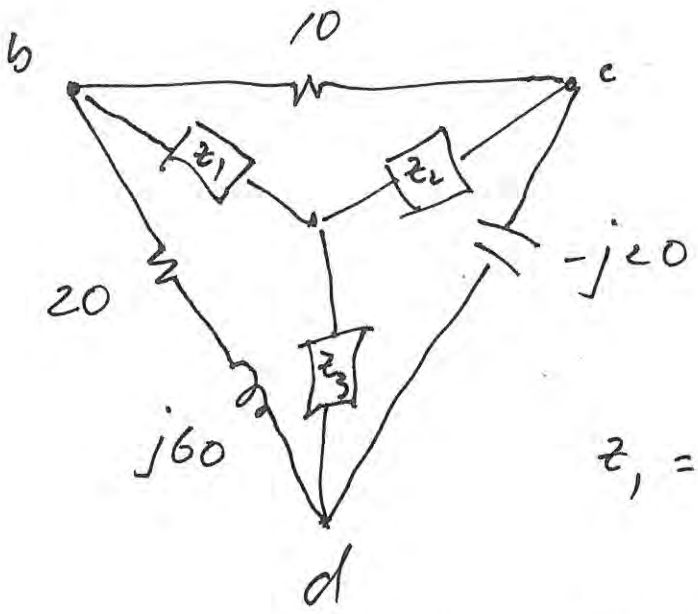
WHICH Δ TO TRANSFORM?

LOOP: $R_T = -j4 + 63.2 + j2.4 + 10$

$= 73.2 - j1.6$

$R_T = 10 - j20 + j60 + 20$

$= 30 + j40$



$$z_1 = \frac{(10)(20+j60)}{R_T} = \frac{200(1+j3)}{30+j40}$$

$$z_2 = \frac{(10)(-j20)}{R_T} = \frac{-j\,200}{30+j40}$$

$$z_3 = \frac{(20+j60)(-j20)}{R_T} = \frac{400(3-j1)}{30+j40}$$

HOW TO CALCULATE – LEAVE IN RECT OR CONVERT TO POLAR?

$$z_1 = \frac{200(1+j3)}{10(3+j4)}\,\frac{(3-j4)}{(3-j4)} = \frac{200\,(3-j4+j9+12)}{10\,(9+16)}$$

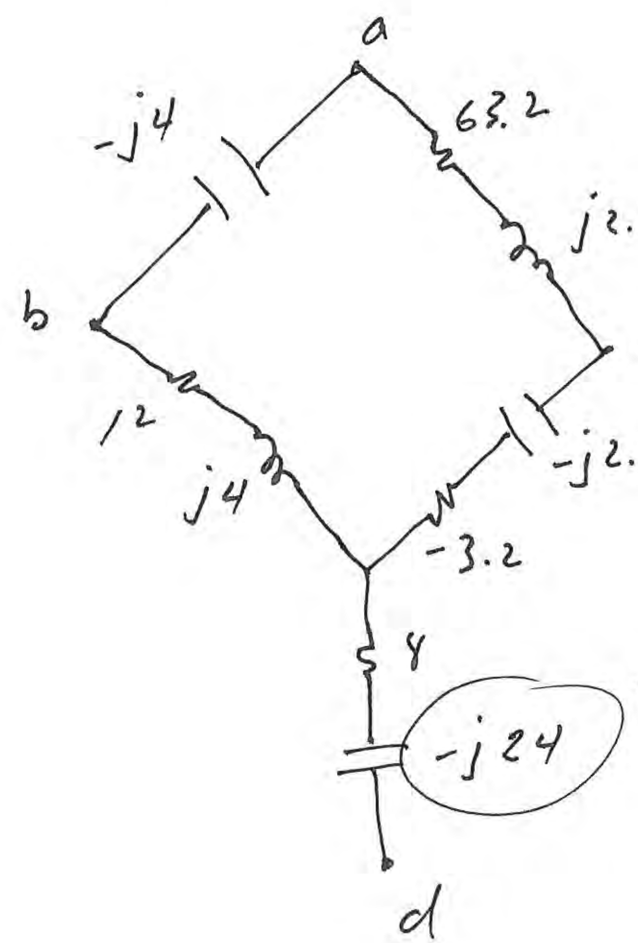
$$z_1 = \frac{200}{250}(15+j5) = 12+j4$$

$$z_2 = \frac{-200(j)(3-j4)}{10(3+j4)(3-j4)} = \frac{-200(4+j3)}{10(25)}$$

$$= -\frac{16}{5} - j\frac{12}{5} = -3.2 - j2.4$$

$$z_3 = \frac{400(3-j1)(3-j4)}{10(3+j4)(3-j4)} = \frac{400(9-j12-j3-4)}{10(25)}$$

$$= \frac{400}{250}(5-j15) = 8 - j24$$

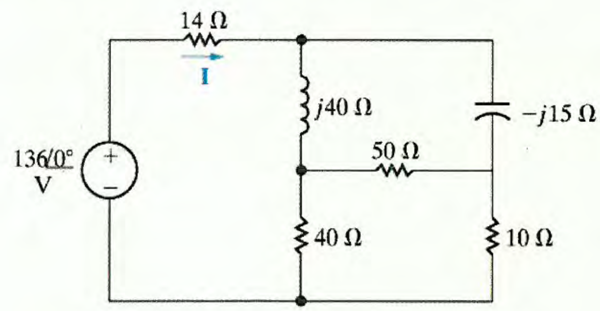


— note typo in fig 9.22

✓ASSESSMENT PROBLEM

Objective 3—Know how to use circuit analysis techniques to solve a circuit in the frequency domain

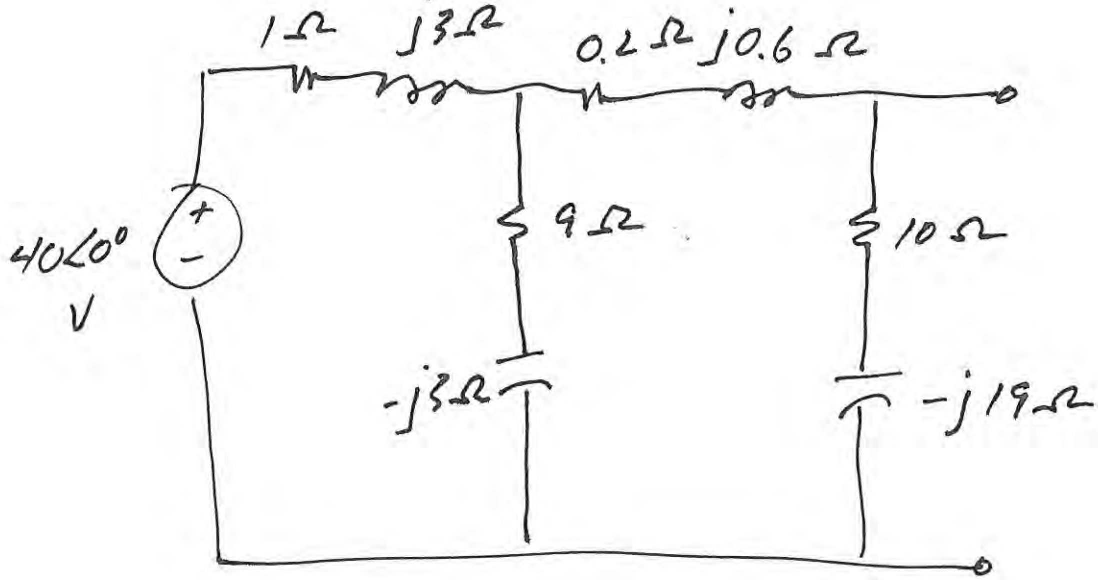
9.9 Use a Δ-to-Y transformation to find the current **I** in the circuit shown.

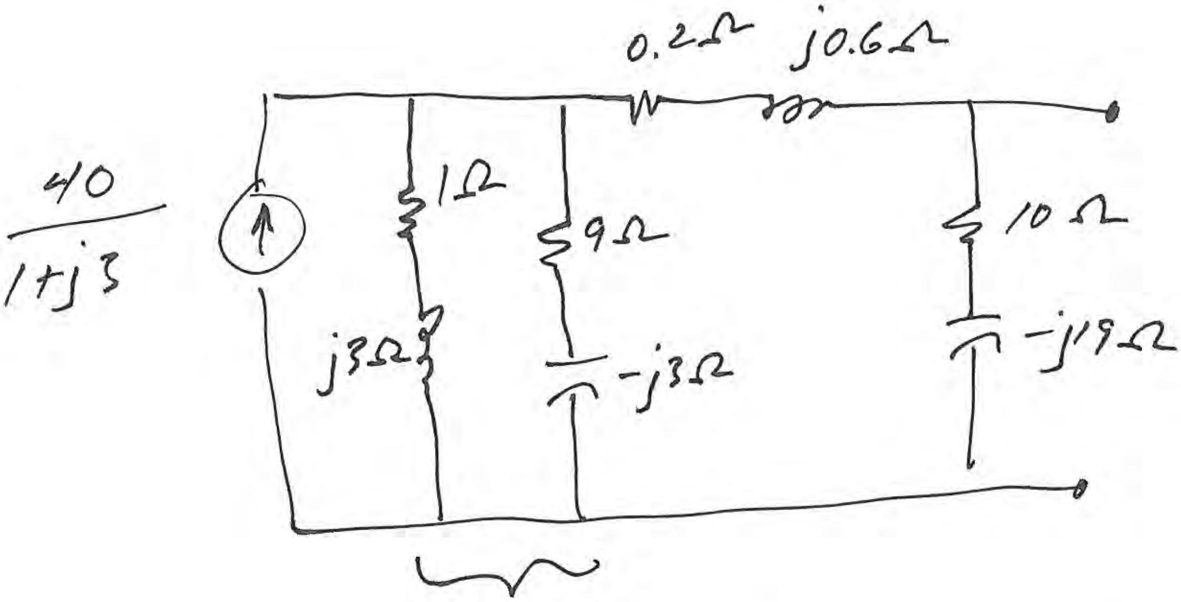


Answer: $\mathbf{I} = 4 \angle 28.07^\circ$ A.

NOTE: Also try Chapter Problem 9.42.

EXAMPLE 9.9 SOURCE TRANSFORMATIONS



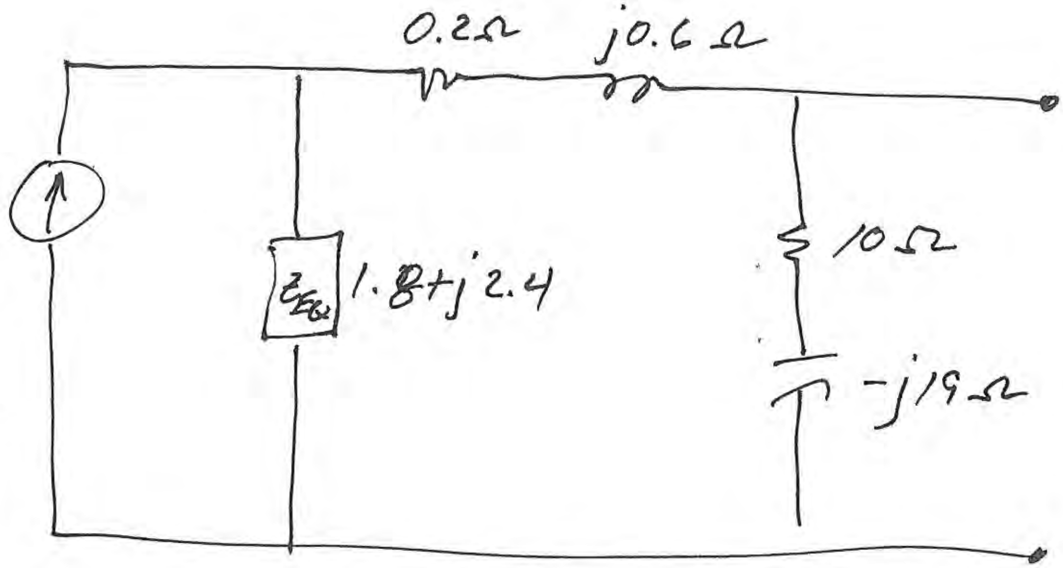


$$z_{EQ} = (1+j3) \parallel (9-j3)$$

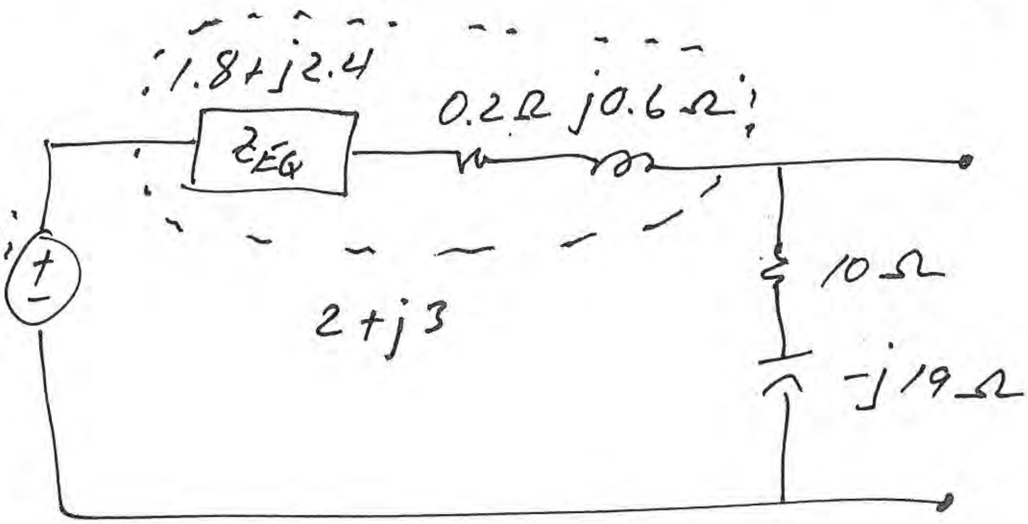
$$= \frac{(1+j3)(9-j3)}{1+j3+9-j3} = \frac{18+j24}{10}$$

$$z_{EQ} = 1.8 + j2.4$$

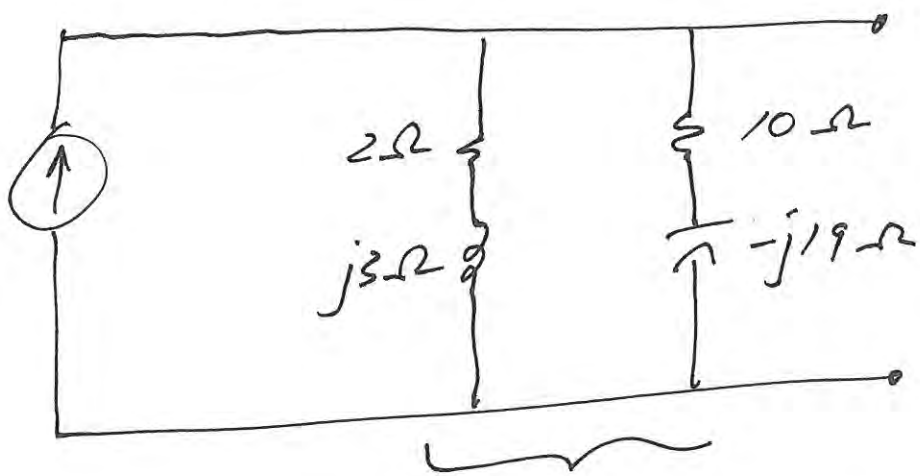
$$\frac{40}{1+j3} = 4-j12$$



$$\frac{40(1.8+j2.4)}{1+j3} = 36-j12$$

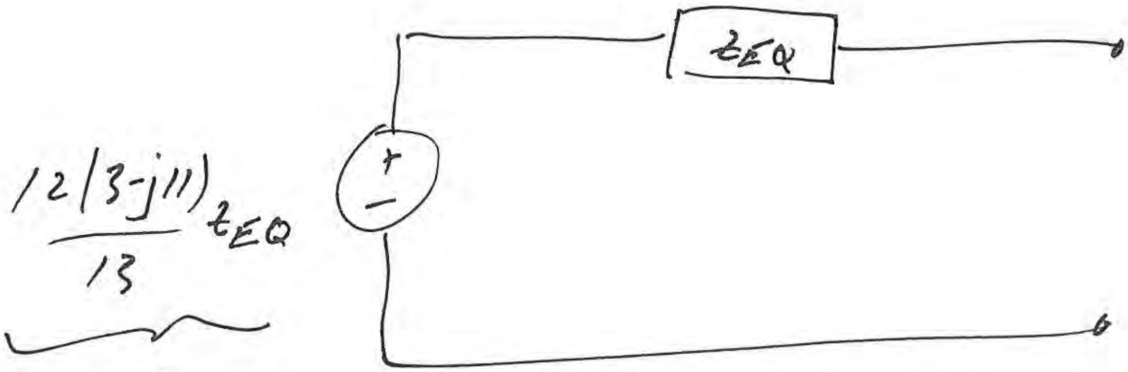


$$\frac{40(1.8+j2.4)}{(1+j3)(2+j3)} = \frac{12(3-j11)}{13}$$

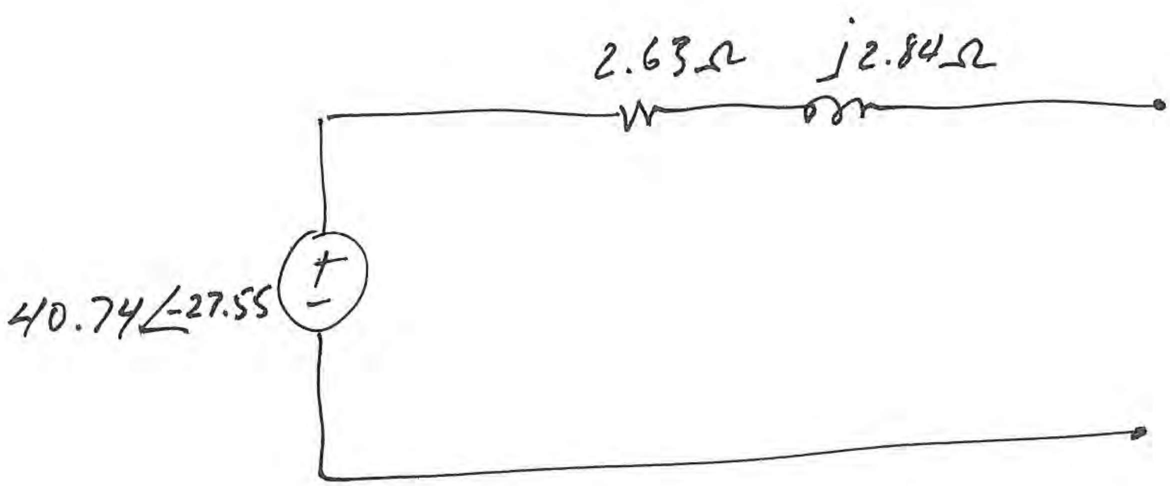


$$Z_{EQ} = \frac{(2+j3)(10-j19)}{2+j3+10-j19}$$

$$Z_{EQ} = \frac{263 + j284}{100} = 2.63 + j2.84\ \Omega$$



$$V = \frac{12(3-j11)}{13}\left(\frac{263+j284}{100}\right) = 36.12 - j18.84\ \Omega$$



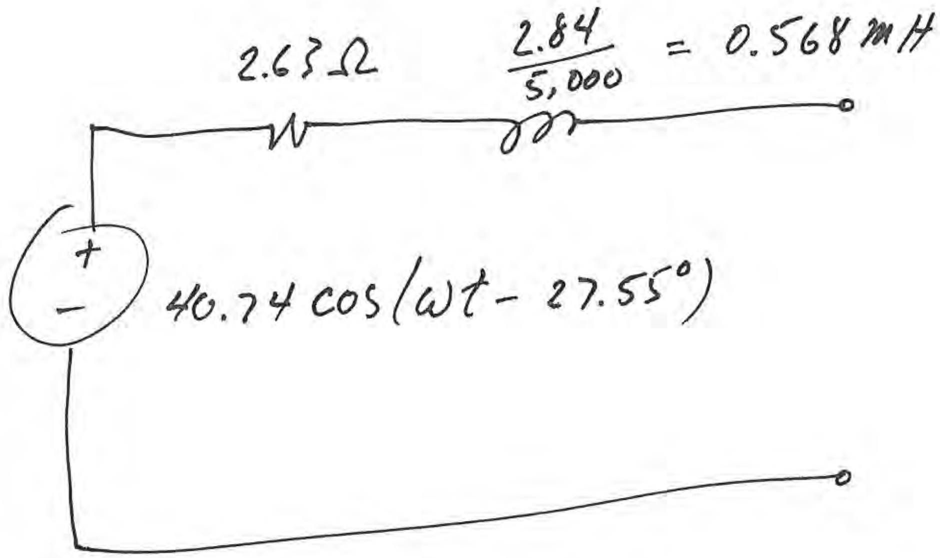
MY TIME DOMAIN CKT CHOSE

$j3\Omega = j\omega L\ \Omega$

WITH $\omega = 5{,}000$ RAD/S

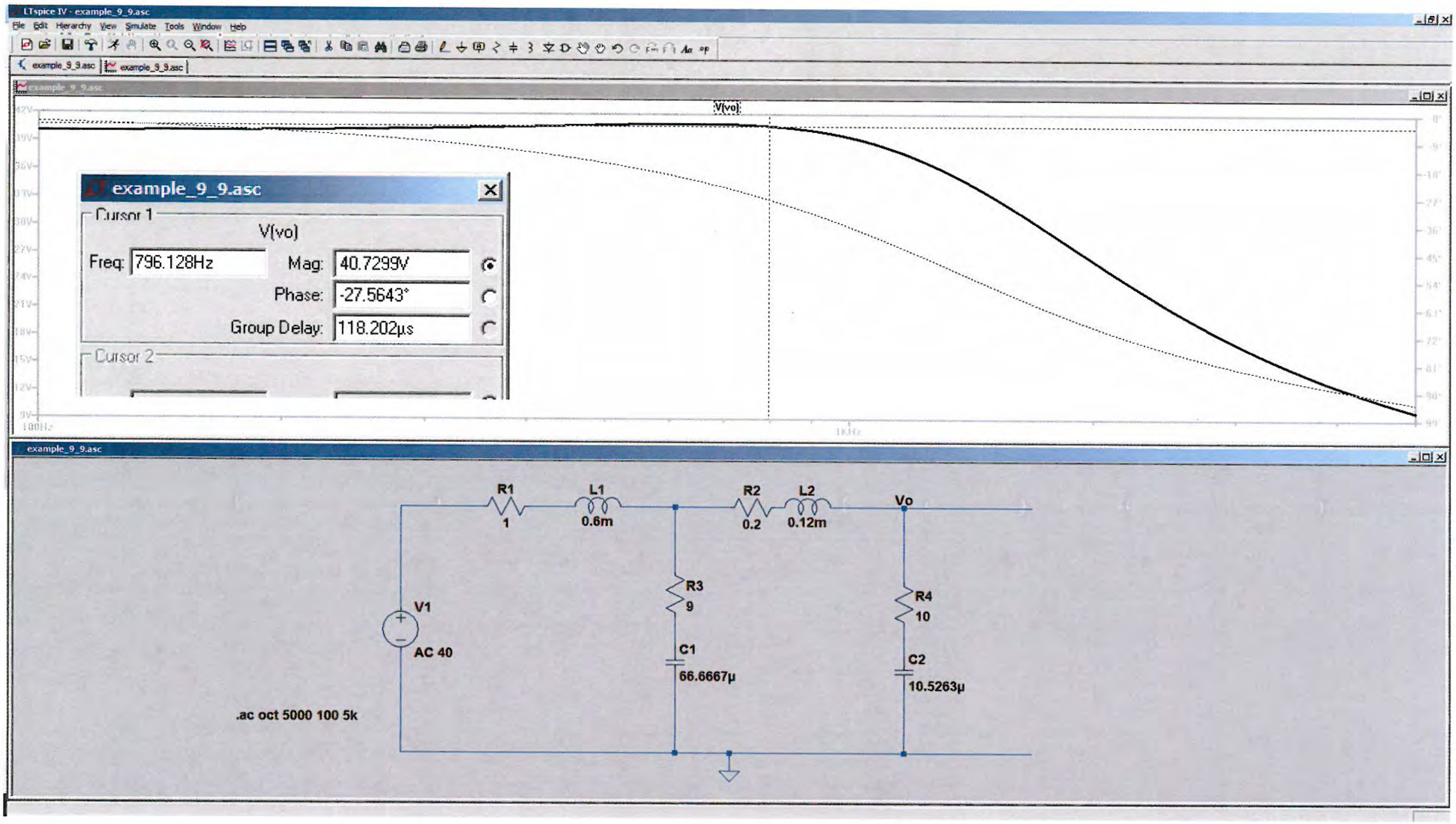
$\Rightarrow j3\Omega = j(5{,}000)(0.6\,mH)$

FINAL TIME DOMAIN EQ CKT IS

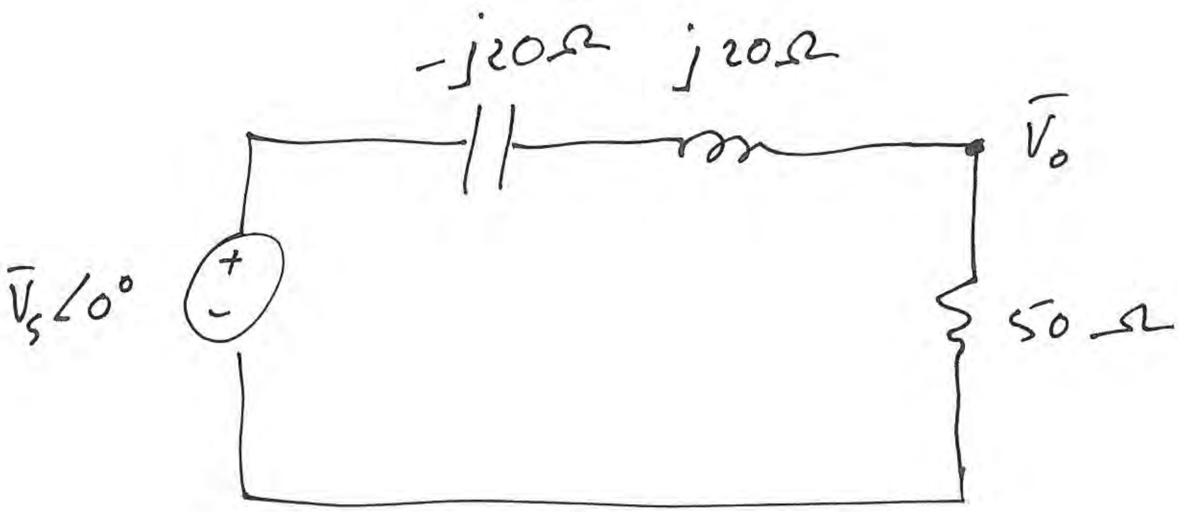


IS THIS VALID FOR ARBITRARY FREQUENCY, ω ?

Example 9.9



LTspice IV - example_9_9.asc
File Edit Hierarchy View Simulate Tools Window Help
example_9_9.asc
V(vo)
example_9_9.asc
Cursor 1
V(vo)
Freq: 796.128Hz
Mag: 40.7299V
Phase: -27.5643°
Group Delay: 118.202µs
Cursor 2
100Hz
1KHz
R1
1
L1
0.6m
R2
0.2
L2
0.12m
Vo
R3
9
C1
66.6667µ
R4
10
C2
10.5263µ
V1
AC 40
.ac oct 5000 100 5k



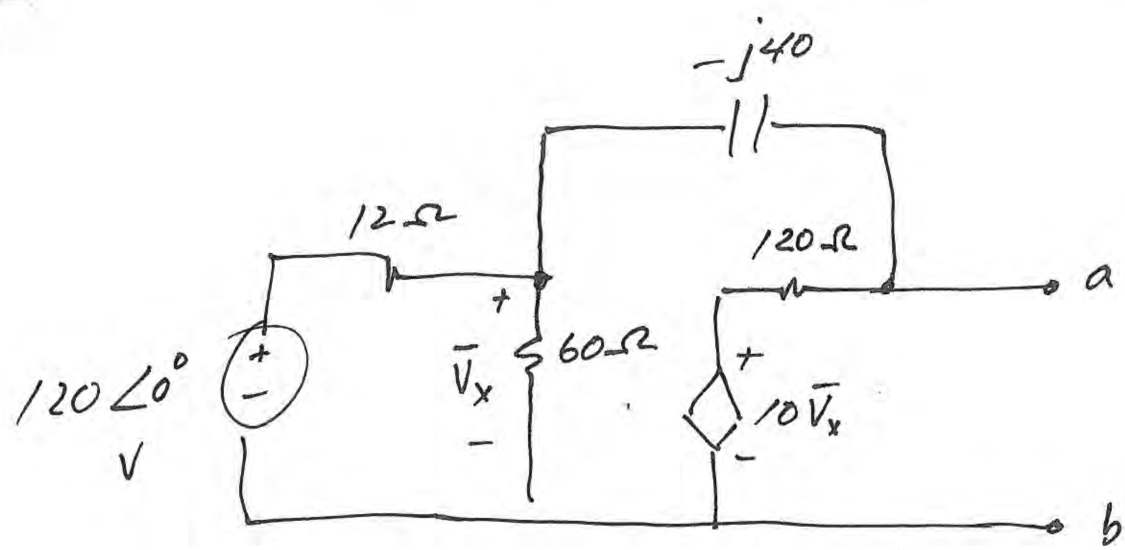
VOLTAGE DIVISION:

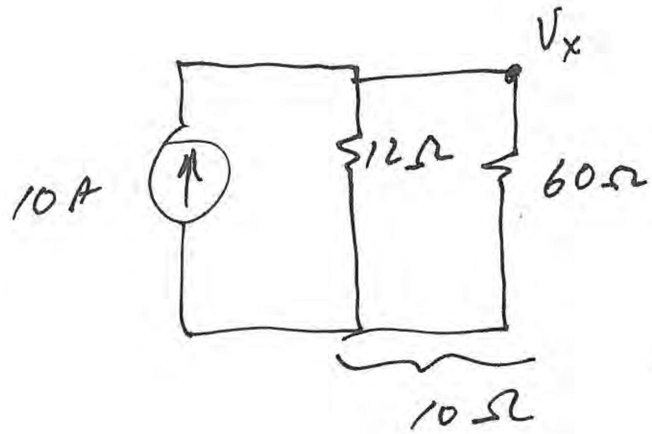
$$\bar{V}_o = \frac{\bar{V}_s\,(50)}{-j20 + j20 + 50} = \bar{V}_s$$

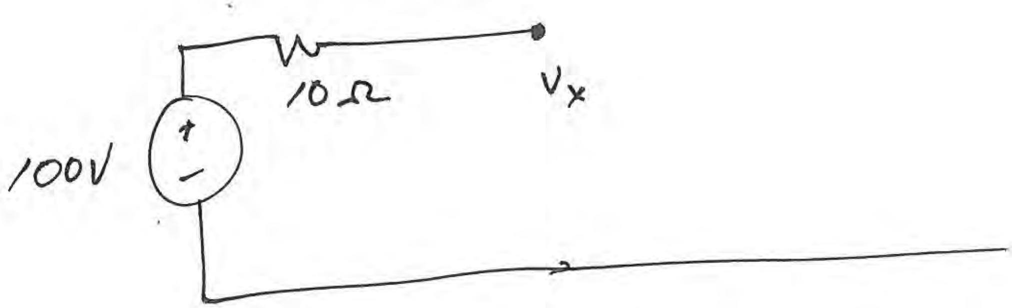
$$\bar{V}_o = \bar{V}_s$$

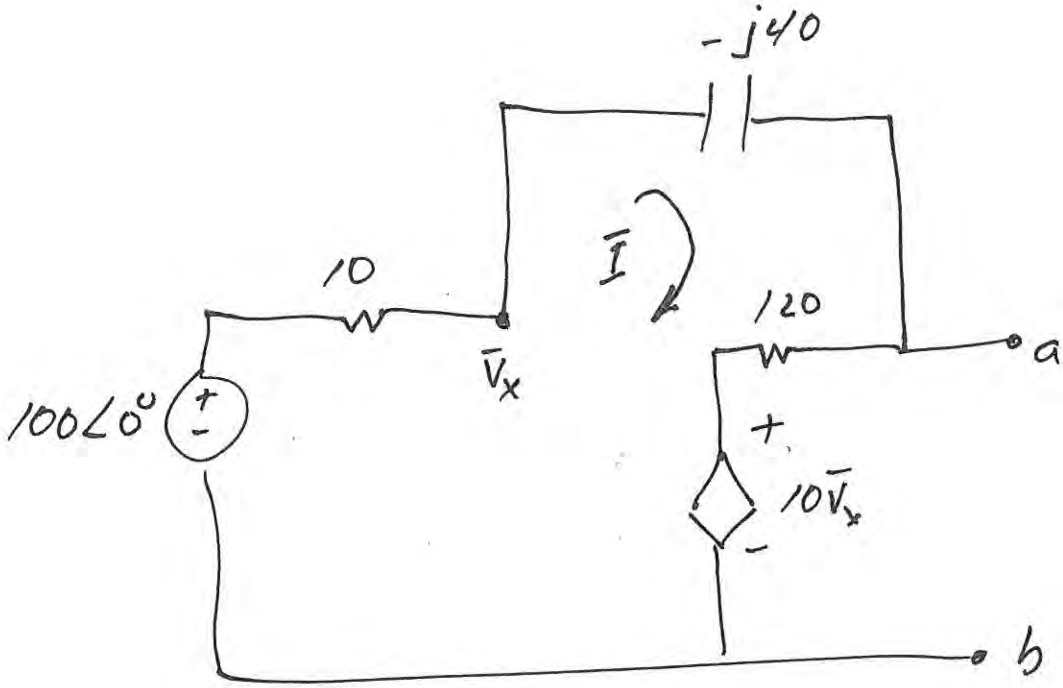
GENERAL SOLUTION?

EXAMPLE 9.10 THÉVENIN EQ IN FREQ DOMAIN









MESH EQ

$$100 - 10\bar{I} - (-j40)\bar{I} - 120\bar{I} - 10\bar{V}_x = 0$$

BUT ~~100~~ $100 - 10\bar{I} = \bar{V}_x$

$$100 - 10\bar{I} + j40\bar{I} - 120\bar{I} - 10(100 - 10\bar{I}) = 0$$

$$100 - 1{,}000 + \bar{I}(-10 + j40 - 120 + 100) = 0$$

$$-900 + \bar{I}(-30 + j40) = 0$$

$$\bar{I} = \frac{-900}{30 - j40}$$

$$\bar{V}_{ab} = \bar{V}_{TH} = 10\bar{V}_x + 120\bar{I}$$

$$\bar{V}_{TH} = 10(100 - 10\bar{I}) + 120\bar{I}$$

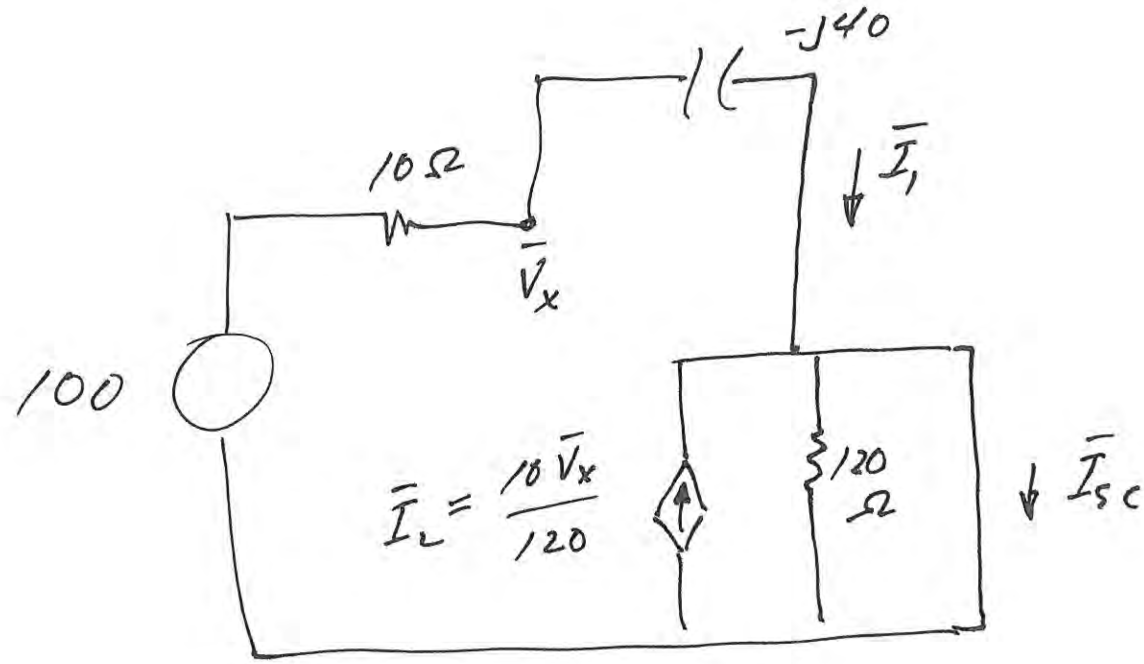
$$= 1{,}000 + 20\bar{I}$$

$$= 1{,}000 + 20\left(\frac{-900}{30 - j40}\right)$$

$$= 1{,}000 - \frac{1{,}800}{3 - j4}$$

$$= \frac{1{,}000(3-j4) - 1{,}800}{3-j4} = \frac{400(3-j10)}{3-j4}$$

$$\bar{V}_{TH} = \frac{400(3-j10)}{3-j4}$$



TWO COMPONENTS OF $\bar{I}_{SC} = \bar{I}_1 + \bar{I}_2$

$$\bar{I}_{SC} = \bar{I}_1 + \frac{10\bar{V}_x}{120} \quad \text{BUT } 100 - 10\bar{I}_1 = V_x$$

$$\text{AND } \bar{I}_1 = \frac{100}{10 - j40}$$

$$\bar{I}_{SC} = \bar{I}_1 + \frac{10}{120}(100 - 10\bar{I}_1)$$

$$= \frac{100}{12} + \frac{2\bar{I}_1}{12}$$

$$\bar{I}_{SC} = \frac{100}{12}\left(1 + \frac{2}{10 - j40}\right)$$

$$= \frac{100}{12}\left(1 + \frac{1}{5 - j20}\right) = \frac{100}{12}\left(\frac{5 - j20 + 1}{5 - j20}\right)$$

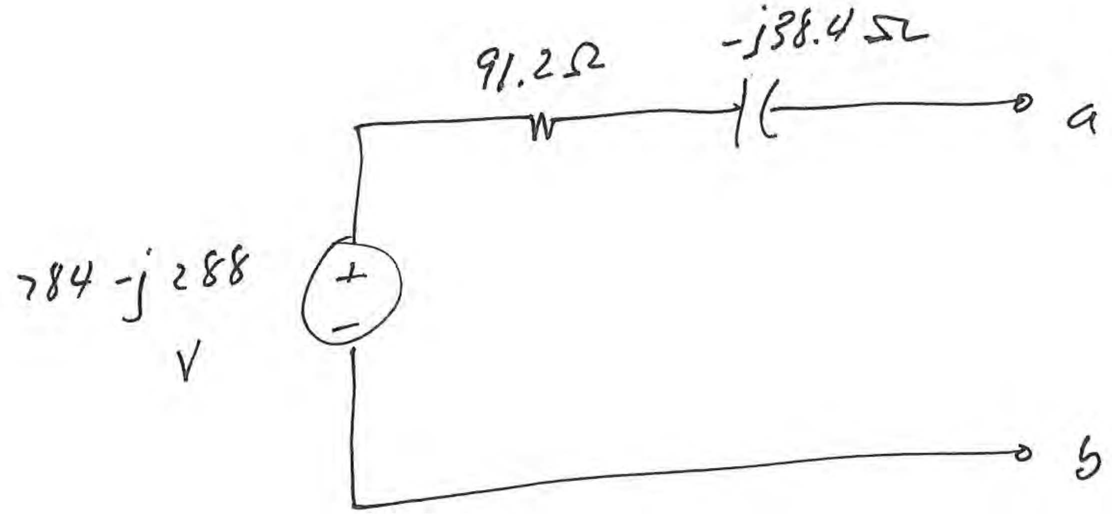
$$\bar{I}_{sc} = \frac{20}{6}\left(\frac{3-j10}{1-j4}\right)$$

$$Z_{TH} = \frac{\bar{V}_{TH}}{\bar{I}_{sc}} = \frac{400(3-j10)}{(3-j4)} \; \frac{6(1-j4)}{20(3-j10)}$$

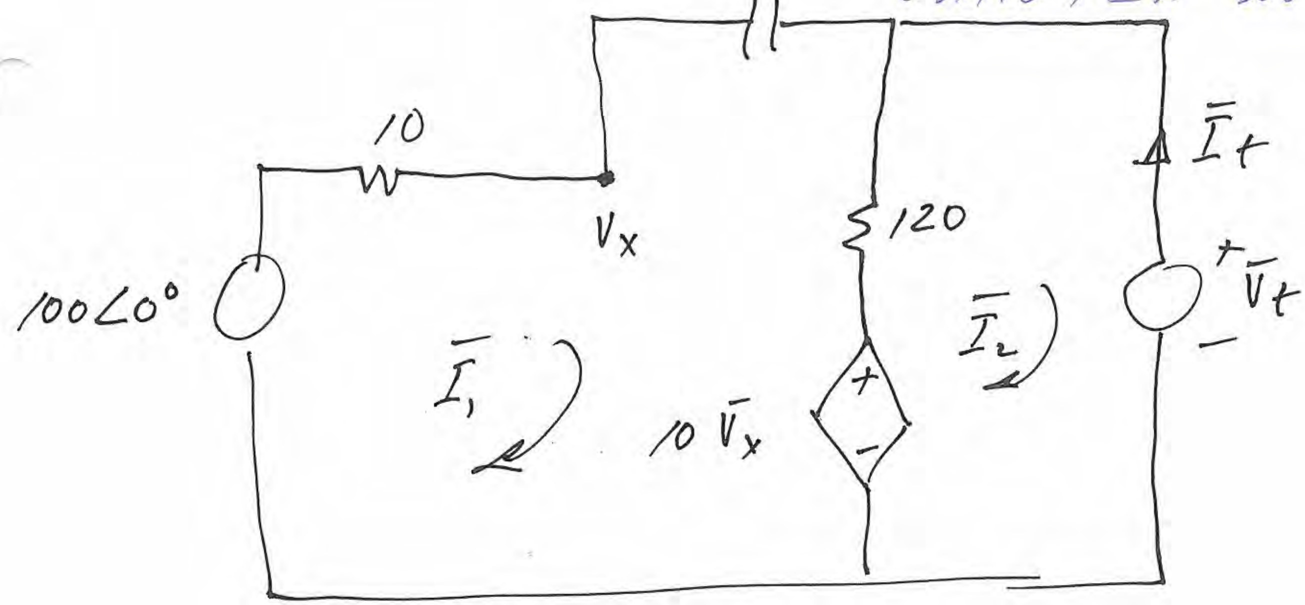
$$= \frac{20 \cdot 6\,(1-j4)}{(3-j4)} \; \frac{(3+j4)}{(3+j4)}$$

$$= \frac{20 \cdot 6}{25}(1-j4)(3+j4)$$

$$Z_{TH} = \frac{120}{25}(19-j8) = 91.2 - j38.4\ \Omega$$



ALTERNATE
USING TEST SOURCE



$$100 - 10\bar{I}_1 - (-j40)\bar{I}_1 - 120(\bar{I}_1 - \bar{I}_2) - 10\bar{V}_x = 0$$

$$10\bar{V}_x - 120(\bar{I}_2 - \bar{I}_1) - V_t = 0$$

BUT $100 - 10\bar{I}_1 = \bar{V}_x$

$$100 - 10\bar{I}_1 + j40\bar{I}_1 - 120(\bar{I}_1 - \bar{I}_2) - 10(100 - 10\bar{I}_1) = 0$$

$$10(100 - 10\bar{I}_1) - 120(\bar{I}_2 - \bar{I}_1) - V_t = 0$$

$$I_t = \frac{\text{DET}\begin{vmatrix} (-30+j40) & 900 \\ 20 & V_t - 1000 \end{vmatrix}}{\text{DET}\begin{vmatrix} (-30+j40) & -120 \\ 20 & 120 \end{vmatrix}}$$

$$D = 120(-30+j40) + 120(20)$$

$$= 120(-30+j40 + 20)$$

$$= 120(-10+j40) = 1{,}200(-1+j4)$$

$$N = (-30+j40)(V_t - 1{,}000) - 18{,}000$$

$$\bar{I}_t = \frac{(-30+j40)(V_t - 1{,}000) - 18{,}000}{1{,}200(-1+j4)}$$

$$\bar{I}_t\, 1{,}200(-1+j4) + 18{,}000$$

$$= (-30+j40)(V_t - 1{,}000)$$

$$\bar{I}_1(-10 + j40 - 120 + 100) + \bar{I}_2(120) = 1{,}000 - 100$$

$$\bar{I}_1(-100 + 120) + \bar{I}_2(-120) = V_t - 1{,}000$$

$$\bar{I}_2 = -\bar{I}_t$$

$$\bar{I}_1(-30 + j40) + \bar{I}_t(-120) = 900$$

$$\bar{I}_1(20) + \bar{I}_t(120) = V_t - 1{,}000$$

$$\bar{I}_t \frac{1{,}200(-1+j4)}{-30+j40} + \frac{18{,}000}{-30+j40} = V_t - 1{,}000$$

$$V_t = \frac{120(-1+j4)}{-3+j4}\bar{I}_t + \frac{1{,}800}{-3+j4} + 1{,}000$$

$$R_{TH} = \frac{120(-1+j4)}{-3+j4} = \frac{120(1-j4)}{(3-j4)}\frac{(3+j4)}{(3+j4)}$$

$$= \frac{120(1-j4)(3+j4)}{25}$$

$$= \frac{120(3+j4-j12+16)}{25}$$

$$= \frac{120(19-j8)}{25}$$

$$= 91.2 - j38.4\ \Omega$$

$$V_{TH} = \frac{1,800}{-3+j4} + 1,000$$

$$= \frac{1,800 + 1,000(-3+j4)}{-3+j4}$$

$$= \frac{-1,200 + j4000}{-3+j4} = \frac{1,200 - j4000}{3-j4}$$

$$= \frac{100(12-j40)(3+j4)}{(3-j4)(3+j4)}$$

$$= \frac{\overset{4}{\cancel{100}}(12-j40)(3+j4)}{\cancel{25}}$$

$$= 4(36 + j48 - j120 + 160)$$

$$= 4(196 - j72)$$

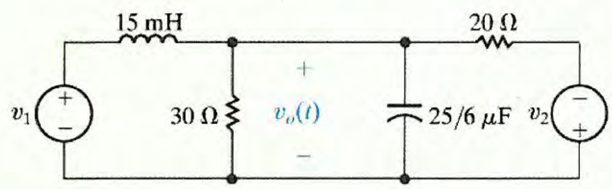
$$V_{TH} = 784 - j288$$

✓ ASSESSMENT PROBLEMS

Objective 3—Know how to use circuit analysis techniques to solve a circuit in the frequency domain

9.10 Find the steady-state expression for $v_o(t)$ in the circuit shown by using the technique of source transformations. The sinusoidal voltage sources are

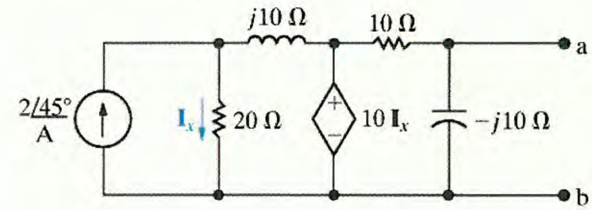
$$v_1 = 240\cos(4000t + 53.13^\circ)\ \text{V},$$
$$v_2 = 96\sin 4000t\ \text{V}.$$



Answer: $48\cos(4000t + 36.87^\circ)$ V

NOTE: Also try Chapter Problems 9.44, 9.45, and 9.48.

9.11 Find the Thévenin equivalent with respect to terminals a,b in the circuit shown.

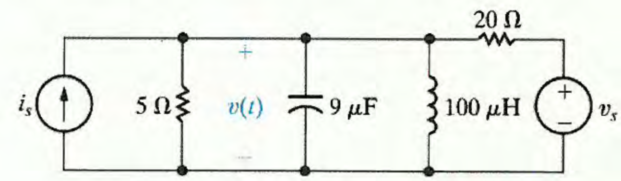


Answer: $V_{Th} = V_{ab} = 10\angle 45^\circ$ V;
$Z_{Th} = 5 - j5\ \Omega$.

✓ ASSESSMENT PROBLEM

Objective 3—Know how to use circuit analysis techniques to solve a circuit in the frequency domain

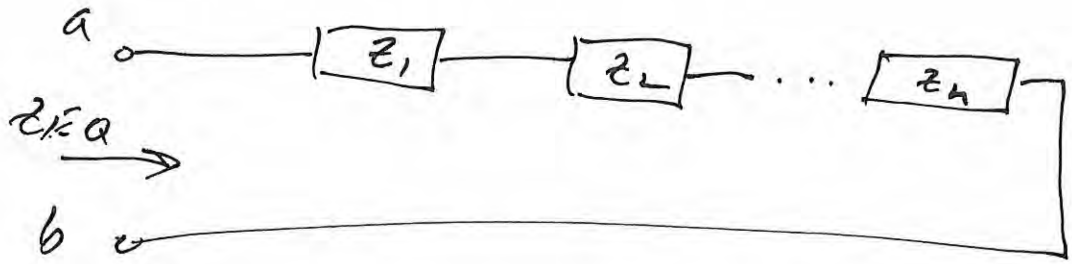
9.12 Use the node-voltage method to find the steady-state expression for $v(t)$ in the circuit shown. The sinusoidal sources are $i_s = 10 \cos \omega t$ A and $v_s = 100 \sin \omega t$ V, where $\omega = 50$ krad/s.



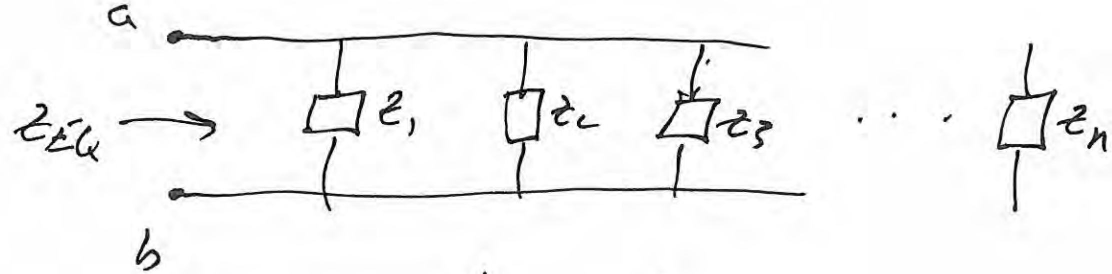
Answer: $v(t) = 31.62 \cos(50{,}000t - 71.57^\circ)$ V.

NOTE: Also try Chapter Problems 9.54 and 9.58.

SIMPLIFICATIONS



$$z_{EQ} = z_1 + z_2 + \cdots + z_n$$



$$z_{EQ}^{-1} = \frac{1}{z_1} + \frac{1}{z_2} + \cdots + \frac{1}{z_n}$$

OLIVER HEAVISIDE

SPECIAL CASE $n=2$: $z_{EQ} = \dfrac{z_1 z_2}{z_1 + z_2}$

EASIER WAY? YES – DEFINE ADMITTANCE

$$Y \equiv \frac{1}{z} = G + jB$$ (~~SIEMANS~~ SIEMENS)

$[Y] = \mho$

$$\Longrightarrow Y_{EQ} = Y_1 + Y_2 + \cdots + Y_n$$

ERNST WERNER VON ~~SIEMANS~~ SIEMENS

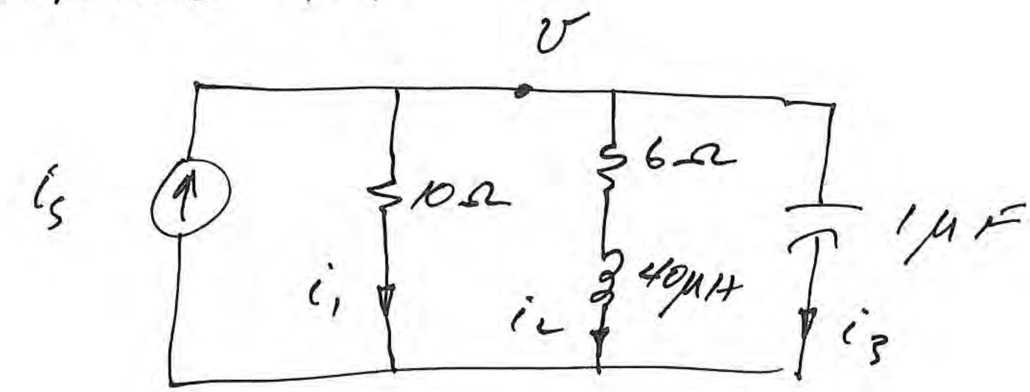
$G \equiv$ CONDUCTANCE

$B \equiv$ SUSCEPTANCE

FOR CAPACITOR $Y = j\omega C$

FOR INDUCTOR $Y = 1/j\omega L$

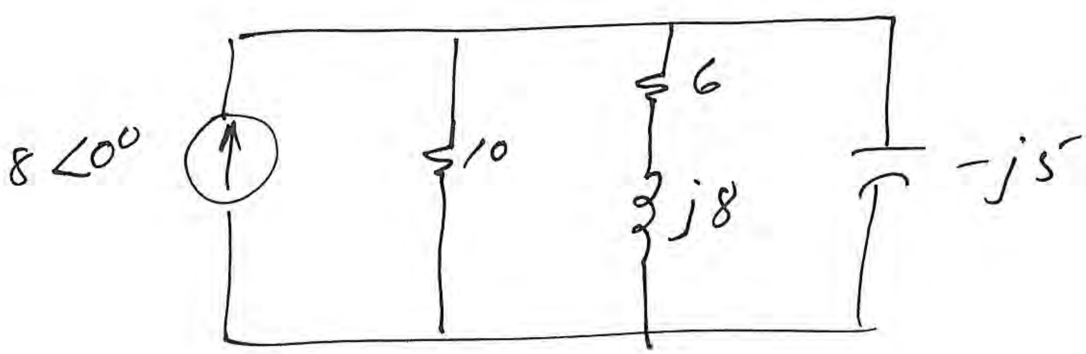
EXAMPLE 9.7



$i_s(t) = 8\cos(200{,}000t)$ A

FIND: i_1, i_2, i_3, v

$\bar{I}_s = 8\angle 0°$ A



$$Y_1 = 0.10$$

$$Y_2 = \frac{1}{6+j8} = \frac{6-j8}{100} = 0.06 - j0.08$$

$$Y_3 = j0.2$$

$$Y_{EQ} = Y_1 + Y_2 + Y_3 = 0.16 + j0.12$$

$$= 0.20 \angle 36.87^\circ$$

$$Z_{EQ} = 5 \angle -36.87^\circ$$

$$\bar{V} = \bar{I}_S Z_{EQ} = 40 \angle -36.87^\circ$$

$$\bar{I}_1 = \frac{\bar{V}}{Z_1} = \frac{40}{10} \angle -36.87^\circ$$

$$\bar{I}_2 = \frac{\bar{V}}{Z_2} = \frac{40 \angle -36.87^\circ}{6+j8}$$

$$= \frac{40 \angle -36.87^\circ}{10 \angle 53.13^\circ} = 4 \angle -90^\circ$$

$$\bar{I}_3 = \bar{V} Y_3 = (40 \angle -36.87^\circ)(0.2 \angle 90^\circ)$$

$$= 8 \angle 53.13^\circ$$